Faint Young Sun Paradox and the Expanding Earth Hypothesis

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Abstract We present a plausible solution to the so-called Faint Young Sun Paradox (FYS-Paradox) within the context of the Expanding Earth Hypothesis (EEH). We show that if — as the current state of the art ITRF observations seem to indicate — the Earth is expanding steadily at a paltry rate of \( \sim +0.36(6) \text{ mm/yr} \) (Shen et al. 2015) and the Earth’s atmosphere is to have a radial vertical height of about one and a half times the Earth’s radius (~ 9860 km) from the Earth’s surface, then, one can (might) explain the presence of liquid water on the Earth’s surface some \( 3.80 - 2.50 \) billion years ago during the Archaean eon when the Sun’s luminosity was about 75% of its current luminosity. Our suggested (proposed) solution makes use of the veritable fact that the albedo of an atmosphere endowed planet will vary in proportion and in response to the expansion (or contraction) of the planet. The Earth system is herein cast as an automatic self-regulating incubator where the auto-self-regulating mechanism is as a result of the solid Earth’s radial expansion. Our findings have significant and serious implications for the conditions obtaining in the early Earth. These conditions obtaining in the early Earth can be used as general sine-qua-non conditions for probing the possibility of life elsewhere in the Universe.

Keywords Expanding Earth – Newton’s Law of Cooling – Radiation Balance Equation – Faint Young Sun Paradox.

1 Introduction

Writing in 1972 in the journal *Science*, renowned American astronomer, cosmologist, astrophysicist, astrobiologist, author, inspirational & one of the greatest science communicators in the history of humankind — Carl Edward Sagan (1934–1996), and his fellow astronomer — George Mullen; brought (for the first time) to the international limelight\(^1\), an apparent paradox concerning the evolution of the Sun and the supposed presence of liquid water on the Earth’s surface (see e.g., Peck et al. 2001; Rosing et al. 1996; Kasting 1993, 1989; Kiehl and Dickinson 1987; Sagan and Mullen 1972). As initially pointed out by Donn et al. (1965), Sagan and Mullen (1972) noted that according to the then just established evolutionary stellar models that describe stars like our own Sun — models that still hold to this day (Gough 1981); the Sun’s energy output (which should have been \( \sim 0.75L_\odot \)) during the Archaean eon\(^2\), \( [i.e. t_A \sim (3.80 - 2.50) \text{ Billion years}] \) should, according to our present wisdom of stellar evolution, have been insufficient to sustain liquid water on the Earth’s surface, the meaning of which is that, contrary, to geological evidence (Peck et al. 2001; Rosing et al. 1996), liquid water should not have been present — hence, the ‘paradox’ of how did a faint young Sun manage to warm-up the Earth to the extend of sustaining liquid water on the Earth’s surface despite the ‘insufficient’ energy budget to do so? Accordingly, the Earth should have had only frozen water, thus, making the prospects of the diversity of life witnessed today, very much remote, if not unlikely. This paleoclimatology paradox is today commonly referred to as the Faint Young Sun Paradox (hereafter FYS-Paradox).

The FYS-Paradox is even more compelling for the planet Mars which we know now to have been covered with oceans

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\(^1\)According to Feulner (2012), it was Donn et al. (1965) who were the first — in recorded literature — to point out the apparent discrepancy between the low solar luminosity predicted for the young Sun and the evidence for liquid water on early Earth. Sagan and Mullen (1972) where the first to bring this problem to the popular attention of the wider scientific community.

\(^2\)Archaean eon, also spelled Archaean eon, is a period in the Earth’s history which began about 4.00 billion years ago with the formation of Earth’s crust and extended to the start of the Proterozoic Eon 2.50 billion years ago. During this time, unicellular organisms are the earliest forms of life that emerged.
for periods of hundreds of millions of years in its early life, with only half of the incoming energy flux of Sunlight of the Earth (e.g., Martens 2016). Proposed solutions to this FYS-Paradox take into account:

1. Greenhouse Effects (see e.g., Sheldon 2006; Eyles and Januszcak 2004; Hessler et al. 2004; Sleep and Zahnle 2001; Rye et al. 1995). In this scenario, enhanced greenhouse effect by carbon dioxide or methane, geothermal heat from an initially much warmer terrestrial core, a much smaller Earth albedo, life developing in a cold environment under a 200 m thick ice sheet, a secular variation in the gravitational constant, etc. According to e.g. Kasting (2004), most of these greenhouse effect models have serious shortcomings: for example, the greenhouse effect from methane appears to be self-limiting, and not enough carbon dioxide is indicated by the geological record to justify a greatly enhanced greenhouse effect in the past. Further, according to Rosing et al. (1996) examination of Archaean sediments appears inconsistent with the hypothesis of high greenhouse concentrations. Rosing et al. (1996) argues that instead, the obtaining moderate temperature range of the Earth’s system through the eons may be explained by a lower surface albedo brought about by less continental area and the ‘lack of biologically induced cloud condensation nuclei’. This, Rosing et al. (1996) says, would have led to increased absorption of solar energy, thereby compensating for the lower solar output.

2. Astrophysical Influences (e.g., Rosing et al. 2010, 1996; Shaviv 2003; Peale 2003). For example, Rosing et al. (1996, 2010) hypothesizes a lower Earth albedo and this owing to considerably less continental area and to the lack of biologically induced cloud condensation nuclei. This would make an important contribution to moderating surface temperature in the Archaean eon. Further, Rosing et al. (2010) suggests that the lower albedo of the early Earth provided environmental conditions above the freezing point of water, thus alleviating the need for extreme greenhouse-gas concentrations to satisfy the FYS-Paradox. In the same vein — albeit, on a different point of departure, our proposed model makes use of a lower albedo of the early Earth and this lower albedo results from a smaller Earth which gradually expands.

3. A combination of both greenhouse and astrophysical influence.

4. A Massive Young Sun (e.g., Martens 2016; Minton and Malhotra 2007). In this scenario, a somewhat more massive young Sun with a large mass loss rate sustained for two to three billion years is assumed. Such a massive young Sun bright enough to keep both the terrestrial and Martian oceans from freezing it thought to resolve the paradox. Martens (2016) finds that a large and sustained mass loss is consistent with the well observed spin-down rate of Sun-like stars, and indeed may be required for it.

Extensive reviews on this subject have been carried out with the most recent being those by Iorio (2015, 2013) and Feulner (2012). The FYS-Paradox not only remains an Open Question, but an active field of research where a solution is much sought (e.g., Airapetian et al. 2016; Marchi et al. 2016; Martens 2016; Iorio 2013, 2015; Wordsworth and Pierrehumbert 2013; Iorio 2015, 2013; Angulo-Brown et al. 2012; Rosing et al. 2010; Minton and Malhotra 2007; Sheldon 2006; Hessler et al. 2004; Kasting 2004; Sleep and Zahnle 2001). In the present endeavour, we shall add a completely new solution to this long-standing and very interesting riddle.

As said, the Archaean eon occurred during the period \( t_A \sim (3.80 - 2.50) \) Byr ago. In order for us to have convenient calculations, we need a single value of \( t_A \) rather than a lower and upper limit. To that end, we shall take the Archaean eon period \( t_A \sim (3.80 - 2.50) \) Byr to be the average of \( (3.80 - 2.50) \) Byr, that is, \( [(3.80 + 2.50)/2 = 3.15 \) Byr and the upper and lower limits (range or ‘error’) of this will be the average of the difference \( [(3.80 - 2.50) \) Byr \( i.e. \) \( (3.80 - 2.50)/2 = 0.65 \) Byr, hence:

\[
t_A = (3.20 \pm 0.70) \times 10^9 \text{yr}. \tag{1}
\]

We shall adopt this value [i.e., equation (1)] as the representative of the Archaean eon, with the upper and lower limits represented by the ‘error’ bars.

Now, from an astrometric standpoint, a viable solution would be that: the Earth may have been much closer to the Sun at \( 95.6\% \) (Iorio 2013) its present day heliocentric distance. This would allow the Earth to receive the required radiation intensity to sustain liquid water on the Earth’s surface and with the progression of time as the Solar luminosity increases, the Earth-Moon system would have to recede from the Sun at just the right rate to maintain steady temperatures. For such a scenario (Iorio 2013), the change in the mean Earth-Moon distance \( \delta r(t)/r(t) \) will have to be related to the change in the Solar luminosity \( \delta L_\odot(t)/L_\odot(t) \), as follows:

\[
\frac{\delta r(t)}{r(t)} = \frac{\delta L_\odot(t)}{L_\odot(t)}. \tag{2}
\]

However (according to, Pitjeva 2012; Pitjeva and Pitjev 2012; Standish 2005; Krasinsky and Brumberg 2004), the currently measured recessional rate of the Earth-Moon system from the Sun of \( (7.00 - 15.00) \) cm/yr, is inadequate.
(Iorio 2015, 2013) to account for this idea of a Closer-Earth-to-the-Sun that would explain the presence of liquid water during the Archaean eon, as this would require a rate as high as \( \sim 180 \text{ cm/yr} \), i.e., \( 30 - 70 \) times the currently measured recessional rate. Given the recessional rate of \( (7.00 - 15.00) \text{ cm/yr} \) and assuming it to be steady, then — it follows that since the Archaean eon, \( \delta r_\oplus(t)/r_\oplus(t) \) must be such that:

\[
\delta r_\oplus(t)/r_\oplus(t) = (3.00 - 1.00) \times 10^{-5}. \tag{3}
\]

Compared to \( \delta L_\odot(t)/L_\odot(t) \), the term \( \delta r_\oplus(t)/r_\oplus(t) \) is four orders of magnitude too small, the meaning of which is that the secular Earth-Moon system drift of \( (7.00 - 15.00) \text{ cm/yr} \) this can not be responsible for the sustenance of the liquid water during the Archaean eon to the present day.

If indeed a closer Earth is the solution to the FYS-Paradox, then, an extra mechanism to that presently pushing the Earth-Moon system away from the Sun would be needed (Nyambuya 2014a,b), the recession of the Earth-Moon system should induce a secular increase in the mean Earth-Moon distance and in-turn, this should lead to an expanding Earth and a contracting Moon. The expansion of the Earth (and contraction of the Moon) is induced by the recession of the Earth-Moon system from the Sun. We shall argue herein that while the recession of the Earth-Moon system from the Sun does not directly solve the FYS-Paradox, the expansion of the Earth provides a real plausible solution via an Earth-expansion induced decrease in the Earth’s albedo.

At this point, we need to point out something very important, especially for the main result of the present reading. The luminosity rate \( \delta L_\odot(t)/L_\odot(t) \) is quoted in the literature as \(+0.25\), i.e., it has a positive value. For the present purposes, we need to ‘correct’ this. The actual value is negative, i.e.:

\[
\delta L_\odot(t)/L_\odot(t) = -0.25. \tag{4}
\]

Why do we say this? Well, the reason is very simple. We know that the luminosity is a measure of the mass loss and losses are represented with a negative sign while mass gain is represented with a positive sign. It is convention that we represent luminosity with a positive sign. When we say that the solar luminosity \( L_\odot = 3.26 \times 10^{26} \text{ W} \), by some unwritten common consensus (convention), we readily understand this to mean an energy loss rate and not energy gain rate. Thus, when we say the luminosity of the Sun as increased by \( 25\% \) since the Archaean eon, in-terms of he actual value, we are talking of a decrease in the actual value since that the luminosity is negative. That is to say, if we are to move say — from \( (Q = -2) \) to \( (Q = -7) \), there has been a decrease in the value of the quantity \( Q \), since \( ‘-2’ \) is greater than \( ‘-7’ \), albeit, in-terms of the magnitude, there has been an increase. It is in on this pedestal of understanding, that the luminosity rate is to have a negative value as has been done in equation (4). Our reason for this reconsideration will become clear when we arrive at equation (38).

Now, here at the ante-penultimete, we need to state that in the present work, we shall demonstrate (and hence propose) a solution under the currently observed recessional rate of the Earth-Moon system of \( (7.00 - 15.00) \text{ cm/yr} \) (Pitjeva 2012; Pitjeva and Pitjev 2012; Standish 2005; Krasinsky and Brumberg 2004) assuming this rate has — through the eons — been steady i.e., one can explain the steady temperatures for the Earth system, hence the presence of the diversity of life seen today on Earth and stretching back to as far as the Archaean eon. In our model, though this recessional rate is not the direct cause of the sustenance of the requisite conditions for the right amount of Solar flux, this rate is just right to give the Earth the right amount of gravitational potential energy to expand; this hypothetical Earth expansion naturally leads to the albedo of the Earth to have just the right values throughout the Earth’s history in-order to maintain steady average global temperatures for the Earth that allow for the existence of liquid water through the eons. If this is correct, it is very humbling to note that the Earth appears to have most of its aspects adjusted so as to allow for the existence of the diversity of life. Further, this should give one an esoteric and very rare sense of privilege to be alive.

The synopsis of this reading is as follows. In the subsequent section, we shall present an exposition of the derivation of the Solar radiation balance equation whereafter in §(3), we give a critic of the derived Solar radiation balance equation and in §(4), we provide an improved Solar radiation balance equation. In §(5), we discuss the expanding Earth hypothesis. In §(6), we bring in a new concept of the effective radius of the Earth. In §(7), we discuss the concept of a changing albedo for a radially expanding Earth. In §(8), from an effective average global temperature of the Earth standpoint, we discuss the model of the Earth with a constant effective average global temperature. In §(9), we present our proposed solution to the FYS-Paradox and lastly, in §(10), we give a general discussion and the conclusion drawn thereof.

### 2 Solar Radiation Balance Equation

As is well known — the temperature of the Earth’s surface has been remarkably constant over geological epochs and this comes mainly from isotopic considerations of Mg/Ca
ratio of foram tests, alkenones\(^3\) and especially \(δ^{18}O\) (e.g., Sigman and Boyle 2000). Even the dramatic cooling during the Ice Age\(^4\) represented a change of only \(\lesssim 1\%\), that is a \(\sim 3.00^{\circ}\)C change in the global average surface temperature, occurring over thousands of years. Seasonal changes in temperature, although large in a particular place, correspond to very tiny changes in mean global temperature. To maintain this long-term temperature stability, the Earth must radiate into space a flux of energy sufficient to just balance the input from the Sun — the meaning of which is that, to a good degree of approximation, the Earth has been, and most probably is, in radiative equilibrium with the Sun’s radiation that it freely receives.

Now, let \(\mathcal{L}_\oplus(t) = 4\pi σ_0 R^2_\oplus(t)T^4_\oplus(t)\) be the Solar luminosity at time \(t\), with \(σ_0\) being the usual Stephan-Boltzmann constant, \(R_\oplus(t)\) the Solar radius at time \(t\), \(T_\oplus(t)\) the Solar temperature at time \(t\); and let \(r(t)\) be the mean distance of a planet from the Sun at time \(t\). The Solar flux total \(F_\oplus(r, t)\) arriving at the spherical shell of radius \(r\) centred about the Solar center is such that:

\[
F_\oplus(r, t) = \frac{\mathcal{L}_\oplus(t)}{4\pi r^2(t)} = σ_0 \left(\frac{R_\oplus(t)}{r(t)}\right)^2 T^4_\oplus(t).
\]

The total Solar energy arriving at Earth per second [Power: \(P_\oplus(t)\)] can be calculated by multiplying \(F_\oplus(r, t)\) by the cross-sectional area (not the total surface area!) of the Earth \(\pi R^2_\oplus(t)\), i.e. the area of Solar beam intersected by the Earth. That is to say, \(P_\oplus(t) = πF_\oplus(r, t)R^2_\oplus(t)\). Not all Solar radiation intercepted by the Earth is absorbed — a good fraction of it is reflected back into space. The fraction of incident Solar radiation reflected is defined as the albedo and denoted by the symbol \(A\), and the fraction absorbed by the Earth at time \(t\) is therefore \([1 – A_\oplus(t)]\). The effective power \(P^\text{abs}_\oplus(t)\) absorbed by the Earth is therefore given by:

\[
P^\text{abs}_\oplus(t) = π[1 – A_\oplus(t)]F_\oplus(r, t)R^2_\oplus(t).
\]

The total energy emitted per unit area is given by \(σ_0 T^4_\oplus(t)\), and the emitting area is the surface area of the Earth, \(4πR^2_\oplus(t)\), therefore, the total energy emitted by the Earth per second is therefore:

\[
P^\text{emit}_\oplus(t) = 4πσ_0 T^4_\oplus(t)R^2_\oplus(t).
\]

Energy balance requires that \((\text{Input} = \text{Output})\) so that when averaged over eons, we will have: \(\{P^\text{abs}_\oplus(t) = P^\text{emit}_\oplus(t)\}\), thus:

\[
4πσ_0 T^4_\oplus(t)R^2_\oplus(t) = π[1 – A_\oplus(t)]F_\oplus(r, t)R^2_\oplus(t).
\]

This is the Energy Balance Equation. It can be solved for the average temperature at which the Earth must emit radiation to bring the energy budget into balance, called the effective temperature \(T_\oplus(t)\) of the planet:

\[
T_\oplus(t) = \left(\frac{[1 – A_\oplus(t)]F_\oplus(r, t)}{4σ_0}\right)^{1/4}.
\]

and this can further be reduced to:

\[
T_\oplus(t) = \left(\frac{1 – A_\oplus(t)}{4}\right)^{1/4} \left(\frac{R^2_\oplus(t)}{r^2_\oplus(t)}\right)^{1/2} T_\oplus(t).
\]

In the subsequent section, we shall on three accounts critic the derivation of this energy balance equation (10).

3 Critic to the Radiation Balance Equation

We have three issues to critic in the above derivation of the effective temperature of the Earth. In the order of their importance here, these there issues are: (a) *Newton (1700)*’s Law of Cooling, (b) the Effective Radius of the Earth and (c) the Emissivity of the Earth, i.e.:

1. *Newton (1700)*’s Law of Cooling: is not taken into account in the derivation of the radiation balance equation. This law (Newton 1700) states that the rate at which a body’s temperature changes is proportional to the difference in temperature of that body with the temperature of its surroundings. Written in differential form, Newton (1700)’s Law translates to:

\[
\frac{dT(t)}{dt} \propto [T(t) – T_{\text{be}}(t)].
\]

Mathematically, equation (11) is convertible into an equation as follows:

\[
\frac{dT(t)}{dt} = -\frac{ckB \mathcal{R} [T(t) – T_{\text{be}}(t)]}{\lambda_N^4 C},
\]

where \(ckB \mathcal{R} /\lambda_N^4 C\) is a constant of proportionality\(^5\) with \(λ_N\) being a parameter (with dimensions of length) that is characteristic of the body under consideration and \((C, \mathcal{A})\) is the

\(^3\)Alkenones are long-chain unsaturated methyl and ethyl n-ketones produced by a few phytoplankton species of the class Prymnesiophyceae (e.g., Marlowe et al. 1984).

\(^4\)The Ice Age is believed to be a period of long-term reduction in the temperature of Earth’s climate, resulting in an expansion of the continental ice sheets, polar ice sheets and mountain glaciers. There are three main types of evidence for ice ages: geological, chemical, and paleontological.

\(^5\)Generally, this constant of proportionality is usually given the symbol \(k\). Here, we decided to decompose it into related constants with \(λ_N\) playing the part of the constant that is characteristic of the body in question.
Table 1 Effective and Actual Temperatures of Mercury, Venus, Earth & Mars: Column (1) – (6) gives the name of the planet, its radius, orbital semi-major axis, its albedo, the actual global average temperature $T_a(t_0)$, the expected global average temperature $T_{pl}(t_0)$ in-accordance with equation (10), and the last column (7) gives the difference $[T_a(t_0) - T_{pl}(t_0)]$ in actual and expected global average temperatures of the listed planets.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Radius ($R_{\oplus}$)</th>
<th>Semi-major axis (1AU)$^a$</th>
<th>Albedo$^b$</th>
<th>$T_a(t_0)$ (K)$^c$</th>
<th>$T_{pl}(t_0)$ (K)</th>
<th>$T_a(t_0) - T_{pl}(t_0)$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.38</td>
<td>0.39</td>
<td>0.12</td>
<td>440.15</td>
<td>433.53</td>
<td>6.62</td>
</tr>
<tr>
<td>Venus</td>
<td>0.95</td>
<td>0.72</td>
<td>0.75</td>
<td>737.15</td>
<td>231.53</td>
<td>505.62</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
<td>0.29</td>
<td>288.15</td>
<td>255.62</td>
<td>32.53</td>
</tr>
<tr>
<td>Mars</td>
<td>3.40</td>
<td>1.52</td>
<td>0.16</td>
<td>208.15</td>
<td>215.99</td>
<td>-7.84</td>
</tr>
</tbody>
</table>


This body’s heat capacity and effective total surface area respectively. The parameters of the constant of proportionality $c_k P/A/\lambda_N^3$, have been carefully chosen for latter convenient purposes. The change in the thermal heat energy $\delta Q_{th}(t)$ is such that $\delta Q_{th}(t) = C dT(t)$: from this it follows that:

$$\varepsilon_{th}(t) = \frac{1}{A} \frac{dQ_{th}(t)}{dt} = -\frac{c_k}{\lambda_N^3} \frac{[T(t) - T_{th}(t)]}{\lambda_N^3},$$

where $\varepsilon_{th}$ is the radiation rate of the thermal energy per unit area. Surely, the planet Earth’s surroundings are at a different temperature to that of the Earth. There surely is no reason whatsoever to not take this into account.

2. Effective Radius of the Earth: The fact that the Earth has an atmosphere is ignored in the traditional calculation presented in §(2). If this is not the case that the Earth’s atmosphere is ignored, then, the effective radiation capture radius $R_{\text{eff}}(t)$ of the Earth in equation (8) will not be equal to the radius of the Earth but will be equal to the radius of the Earth plus the size $\mathcal{H}_{\oplus}(t)$ of the Earth atmosphere — i.e., $R_{\text{eff}}(t) = R_{\oplus}(t) + \mathcal{H}_{\oplus}(t)$. Therefore, we need to make a correction for this. However, if NLC is not taken into account as pointed out above, this correction of the effective radius will not change anything.

What is the value of $\mathcal{H}_{\oplus}(t)$? The truth is that, height of the Earth’s atmosphere is not well known and most often the Kármán line, at $\sim 100$ km, [i.e., 0.0157 $R_{\oplus}(t_0)$] is often used as the border between the atmosphere and outer space. Atmospheric effects become noticeable during atmospheric reentry of spacecraft at an altitude of around $\sim 120$ km. This definition is accepted by the Fédération Aéronautique Internationale$^6$ (FAI). In §(9), we deduce that — in-accordance with the model that we shall put forward, if the Earth has maintained steady temperatures over the eons, then, the present day height will have to be $[\mathcal{H}_{\oplus}(t_0) \sim 3800$ km]. If this is indeed the case, it would mean that the region $[120 \text{ km} < \mathcal{H}_{\oplus}(t_0) \leq 3800 \text{ km}]$ must be a very thin atmosphere that is difficult to detect via traditional means available for thick atmospheres.

3. Emissivity of the Earth: The emissivity, $\varepsilon_{\oplus}$ of the Earth is assumed to he unity (cf., Jin and Liang 2006; Méndez and Rivera-Valentín 2017). Taken to the letter, this is obviously not correct because emissivity is defined as the ratio of the energy radiated from a material’s surface to that radiated from a blackbody (a perfect emitter) at the same temperature, wavelength and under the same viewing conditions. This ratio varies from 0 to 1, with ($\varepsilon = 1$) for a perfect blackbody and ($\varepsilon = 0$) for a perfect absorber. The emissivity is dependent on the type of surface and many climate models set the value of the Earth’s emissivity to 1. However, a more realistic value is $\sim 0.96$ (e.g., Jin and Liang 2006; Méndez and Rivera-Valentín 2017). With the emissivity correction, we will have:

$$T_{\oplus}(t) = \left(1 - \frac{A_{\oplus}(t)}{4\varepsilon_{\oplus}(t)}\right)^{1/4} \left(\frac{R_{\oplus}(t)}{r_{\oplus}(t)}\right)^{1/2} T_{\oplus}(t).$$

Because the actual emissivity is close to unity and appears to be a constant and may have maintained this value since the Archaean eon, this correction is of not much use here or any consideration one may think of. We only presented this for the sack of completeness and nothing more or less.

4 Remedy to the Radiation Balance Equation

From the critique given above — for radiation balance equation, there are two issues worthy of the endeavour to fix. These are (1) to incorporate NLC and (2) to take into account the effective radius as highlighted, where in this correction, one has to take into account the Earth’s atmosphere in-order for an effective radius radius of the Earth. If we are to take into account Newton (1700)’s Law of Cooling, and the fact that the Earth has an atmosphere that does capture some of the incoming Solar radiation, it follows that for the effective power $P_{\text{abs}}(r, t)$ absorbed by the Earth — instead of it being given by (6), it will be given by:

$^6$Founded on Saturday 14 October 1905 and headquartered in, Lausanne — Switzerland, the Fédération Aéronautique Internationale, is the World Governing Body for air sports. The FAI maintains world records for aeronautical activities including ballooning, aeromodeling, and unmanned aerial vehicles; and also for human spaceflight. Official Website: https://www.fai.org/
The difference between equation (6) and (15) is the effective radius, where we have replaced $R$ in equation (6) with $\mathcal{R}^\text{max}$. For the total energy emitted per unit effective surface area of the Earth, we will replace in equation (7) $\mathcal{R}_{\oplus}(t)$ with $\mathcal{R}_{\text{eff}}(t)$ and add to this the Newtonian cooling term, i.e.: $4\pi c_k T_{\text{bg}}(t) - T_{\oplus}(t)\mathcal{R}^\text{max}(t)/\mathcal{A}_{\oplus}$ where $T_{\oplus}(t)$ is the new effective temperature of the Earth; so that the new energy balance will now be given by:

$$
\frac{4\pi c_k T_{\text{bg}}(t) - T_{\oplus}(t)\mathcal{R}^\text{max}(t)}{\mathcal{A}_{\oplus}}
$$

$$
= \pi [1 - A_{\oplus}(t)]\mathcal{F}_{\oplus}(r, t)\mathcal{A}^2_{\text{max}}(t).
$$

(16)

As usual, the term representing the radius of the cross-section of the Earth which this time is $\mathcal{R}_{\text{eff}}(t)$, cancels off so that equation (16) reduces to:

$$
4\pi c_k T_{\text{bg}}^4(t)\mathcal{R}^2_{\text{max}}(t) + \frac{4\pi c_k T_{\text{bg}}(t) - T_{\oplus}(t)\mathcal{R}^\text{max}(t)}{\mathcal{A}_{\oplus}}
$$

$$
= \pi [1 - A_{\oplus}(t)]\mathcal{F}_{\oplus}(r, t)\mathcal{A}^2_{\text{max}}(t).
$$

(17)

With $\mathcal{R}_{\text{eff}}(t)$ cancelling out, it would appear as though our effort to revise the effective radius — *while just, logical, necessary and correct* — has not paid off since this has just cancelled out leaving no clearly visible trace of $\mathcal{R}_{\text{eff}}(t)$ in our midst. As one will come to this full realization soon that, this is not correct. The effective radius $\mathcal{R}_{\text{eff}}(t)$ will come in because of the Newtonian cooling term that we have just added, otherwise, if this Newtonian cooling term was not present, the effective radius correction would yield no new fruit.

Now, this equation (17), instead of it reducing to equation (14), it leads us to the new equation:

$$
T_{\oplus}'(t) = \frac{C_k T_{\text{bg}}(t) - T_{\oplus}(t)}{\lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^4(t)}
$$

(18)

where $T_{\oplus}(t)$ is as defined in equation (10). Assuming that $[c_k T_{\text{bg}}(t) - T_{\oplus}(t)] \ll \lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^4(t)$, it follows that to first order approximation, equation (18) will reduce to:

$$
T_{\oplus}'(t) = T_{\oplus}(t) \left[ 1 - \frac{c_k T_{\text{bg}}(t) - T_{\oplus}(t)}{4\lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)} \right].
$$

(19)

and upon making $T_{\oplus}'(t)$ the subject of the formula in equation (19), we will have:

$$
T_{\oplus}'(t) = \left( \frac{1 + \frac{c_k T_{\text{bg}}(t) - T_{\oplus}(t)}{4\lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)}}{1 + \frac{c_k T_{\text{bg}}(t) - T_{\oplus}(t)}{4\lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)}} \right) T_{\oplus}(t).
$$

(20)

Again, assuming that $[c_k T_{\text{bg}}(t) - T_{\oplus}(t) \ll \lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)]$ and that $[c_k \ll \lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)]$, it follows that to first order approximation, equation (20), will reduce to:

$$
T_{\oplus}'(t) = T_{\oplus}(t) \left[ 1 + \frac{c_k T_{\text{bg}}(t) - T_{\oplus}(t)}{4\lambda^3_{\oplus} c_k \mathcal{A}_{\oplus} T_{\text{bg}}^3(t)} \right].
$$

(21)

Equation (21) is now sought for equation that now replaces equation (14) as the equation that gives us the effective temperature of the Earth (or any similar planetary system). Using the extra-term highlighted in equation (21), one can justify why the average global temperatures are different from $T_{\oplus}(t)$. This reason can be attributed to the surroundings. It is not the intention of the present reading to investigate this matter. We will leave this matter for another reading. All we wanted was to demonstrate that the effective radius of an atmosphere endowed planet is not to be taken as the solid radius of the planet. As will be demonstrated, when it comes to the stability of the Earth’s average global temperature over the eons, there is need to take into account the size of a planet’s atmosphere in the calculation of the Earth’s (planet’s) radiation capture radius. In the next section, we will conduct a brief exposition of the expanding Earth hypothesis. This hypothesis is key to our steady average global temperature model of the Earth.

5 Expanding Earth Hypothesis

The great and renowned German polar researcher, geophysicist and meteorologist — Alfred Lothar Wegener (1880–1930), was the first to propose (suggest) the *polemical* idea of an *Expanding Earth*. This *all-daring hypothesis*, he arrived at upon noticing that the different continental landmasses of the Earth (i.e., continental plates) almost fit *hand-in-glove* together like a *perfect* jigsaw puzzle, thus, Wegener (1912a,b) siezed the moment and proposed (suggested) that these continents are slowly drifting around the Earth, and, as to the cause of this movement, he further proposed that it was as a result of the solid Earth expanding radially outward on a globally scale. Wegener (1912a,b)’s ideas where very attractive to the insatiably curious and inquisitive minds (e.g., Carey 1975; Jordan 1969; Beck 1961;
Cox and Doell 1961; Hixon 1920) and at the same time, so controversial (Creer 1965; Dearnley 1966; Egyed 1961; Heezen 1962), so much that they where not widely accepted until the 1950s, when numerous discoveries such as palaeomagnetism provided strong support for continental drift, and thereby a substantial basis for today’s model of plate tectonics (e.g., Dearnley 1966). While the idea of continental drift was accepted and stands today as the Chief-Corner-Stone of Modern Geophysics, the idea of an expanding Earth was ferociously rejected and is still rejected to this day (see e.g., Edwards 2016; Sudiro 2014; Burša and Šidlíčkovský 1984)

Be that as it may — while the idea of an expanding Earth stands vehemently rejected today by the majority of scientists, we shall demonstrate herein that this idea does offer an interesting solution to this FYS-Paradox. In the light of the advocates of an expanding Earth hypothesis, evidence of an expanding Earth appears to be emerging in the ‘not to distant horizons’ of observational science (Shen et al. 2015,b, 2011; Xu and Sun 2014; Wu et al. 2011; Chen 2000; Gerasimenko 1996, 1997, 1993). At least effort to answer the question of whether or not the Earth is expanding is being made.

Wu et al. (2011) have made the first such direct measurements using data from the International Terrestrial Reference Frame (ITRF). The ITRF is a fundamental datum for precision orbit tracking, navigation, and global change monitoring which combines data from Satellite Laser Ranging (SLR), Very Long Baseline Interferometry (VLBI), Global Positioning System (GPS), and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), its origin is currently realized by the single technique of SLR. To the chagrin of the advocates of an expanding Earth hypothesis, Wu et al. (2011)’s findings are anything but negative! They (Wu et al. 2011) find that, the mean radius of the Earth is not changing to within 1σ-level measurement uncertainty of 0.20 mm/yr. On the ‘good’ side, Shen et al. (2015, 2011) claim to have found evidence of an expanding Earth.

On the positive — just like Wu et al. (2011), the researchers Shen et al. (2011) used the ITRF-2008 space-geodetic data recorded at stations distributed globally (which includes GPS, VLBI, SLR, and DORIS), covering a period of more than ~10 years. From this, Shen et al. (2011)’s calculations show that the Earth is expanding at a rate of ~ +0.24 ± 0.04 mm/yr. From the Archaean eon to the resent day, this gives a change in the Earth radius of:

$$\frac{\delta R_{\oplus}(t_0)}{R_{\oplus}(t_0)} = +0.10 \pm 0.02,$$

that is to say, the Earth has undergone a 10% radial growth since the Archaean eon, if the work of Shen et al. (2011) is to be believed.

Furthermore, based on the Earth Gravitational Model 2008, Shen et al. (2011) finds that the secular variation rates of the second-degree coefficients estimated by SLR and Earth mean-pole data, the principal inertia moments of the Earth and in particular their temporal variations, which they determined, they find a simple mean value of the three principal inertia moments is gradually increasing and this clearly demonstrates that the Earth is indeed expanding, at least over the recent decades, and using this data, Shen et al. (2011) says it [data] shows that the Earth is expanding at a rate ranging from ~ +0.17 ± 0.02 mm/yr to ~ +0.21 ± 0.02 mm/yr, which coincides with the space geodetic evidence. Hence, based on both space geodetic observations and gravimetric data, Shen et al. (2011) concludes that the Earth has been expanding at a rate of ~ +0.20 mm/yr over the recent decade.

In a subsequent and lasted study, Shen et al. (2015) finds an even more favourable result for the expanding Earth. According to Shen et al. (2015), this time using the ITRF-2008 data spanning 20 years, i.e., space-geodetic data recorded at globally distributed stations over solid land, they (Shen et al. 2015) revised their previous estimate of ~ +0.24 ± 0.05 mm/yr. They (Shen et al. 2015) find that from their new two decades of satellite altimetry observations that this data demonstrates that the sea level is raising at a rate of ~ + 3.20 ± 0.40 mm/yr, of which ~ +1.80 ± 0.50 mm/yr is attributed to ice melting over land. Further, Shen et al. (2015) finds that oceanic thermal expansion due to the temperature increase in recent half century is ~ +1.00 ± 0.10 mm/yr.

To this sea level rise observation by altimetry, Shen et al. (2015) points out that this is not balanced by the ice melting and thermal expansion phenomenon, and as such, they take this as an open problem in their study. However, Shen et al. (2015) infer from their studies that the oceanic part of the Earth is expanding at a rate about +0.40 mm/yr and, in conclusion, when combining the expansion rates of land part and oceanic part, they (Shen et al. 2015) find that — at least, over the last two decades or so — the Earth has been expanding at a rate of ~ +0.35 ± 0.47 mm/yr. Additionally, Shen et al. (2015) say that if the Earth expands at this rate, then the altimetry-observed sea level rise can be well explained. The Earth expansion rate of ~ +0.35 ± 0.47 mm/yr (Shen et al. 2015) is 145% (nearly one and a half times) larger than ~ +0.24 ± 0.05 mm/yr (Shen et al. 2015), thus a welcome improvement for they that advocate for an expanding Earth.

From Shen et al. (2015,b, 2011)’s recent work, it appears that those that had long written an obituary of the expanding Earth hypothesis may have to retract them and halt all the efforts that where currently under-way to complete the process by writing the epitaph. In an email private communication — Professor Wenbin Shen (2018), has said that they have improved their results where they now find an Earth expansion
rate of $\sim +0.36 \pm 0.06 \text{ mm/yr}$, which is a weighted result of the land area expansion rate ($\sim +0.45 \pm 0.05 \text{ mm/yr}$) and sea level raise ($\sim +0.32 \pm 0.09 \text{ mm/yr}$). These results—Professor Wenbin Shen (2018), say they are expected to be published in the near future. Thus, hereafter, we shall quote the result $\sim +0.36 \pm 0.06 \text{ mm/yr}$ for Shen et al. (2015) as the present day Earth expansion rate.

6 Solid Earth Radius and Effective Earth Radius Relation

In preparation for the next section where we shall seek a relation between a planet’s change in albedo to its changing solid radius, we shall here seek a relation between the a planet’s solid radius to the planets effective radius. For this, we shall use the general formula for calculating the mass of a planet’s atmosphere. The mass, $m_{\text{atm}}$ of the Earth’s atmosphere (see e.g., Jacob 1999, p.13) is related to Earth global mean surface pressure, $P_{\oplus}(t)$, and the Earth mean surface gravitational acceleration, $g_{\oplus}(t)$, by the following formula:

$$m_{\text{atm}} = 4\pi R_{\oplus}^2(t)P_{\oplus}(t) \over g_{\oplus}(t).$$

(23)

Given that: $[R_{\oplus}(t_0) \simeq 6.40 \times 10^6 \text{ m}]$, $[g_{\oplus}(t_0) \simeq 9.80 \text{ m/s}^2]$ and $[P_{\oplus}(t_0) = 101.325 \text{ kPa}]$, one obtains for the mass of the Earth’s atmosphere ($m_{\text{atm}} \simeq 5.80 \times 10^{18} \text{ kg}$), where here-and-after, $t_0$ represents time in the present epoch — i.e., time since the formation of the Earth-Moon system. This result ($m_{\text{atm}} \simeq 5.80 \times 10^{18} \text{ kg}$), is the widely accepted mass of the Earth’s atmosphere (see e.g., Trenberth and Smith 2005; Jacob 1999; Verniani 1966).

Now, assuming that the Earth follows the Idea Gas Law (this assumption has been used by e.g., Trenberth and Smith 2005; Jacob 1999; Verniani 1966), it follows that:

$$P_{\oplus}(t) = \frac{3}{2} \frac{g_{\oplus}(t)k_B T_{\oplus}}{g_{\oplus}(t)\mu_{\oplus} m_H},$$

(24)

where $k_B, T_{\oplus}, \mu_{\oplus}$ and $m_H$ are the Boltzmann constant, the global mean temperature of the Earth, the mean number of particle per unit molecule of the gas and the mass of the Hydrogen atom, respectively. The mean density density of the Earth’s atmosphere $\rho_{\oplus}$:

$$\rho_{\oplus} = \frac{3m_{\text{atm}}}{4\pi (R_{\text{eff}}^3(t) - R_{\oplus}^3(t))}.$$

(25)

Substituting $\rho_{\oplus}(t)$ as given in equation (25) into equation (24), and then — substituting the resultant expression into equation (23), and thereafter rearranging the resulting expression, one obtains the following:

$$\frac{\delta R_{\oplus}(t)}{R_{\oplus}(t)} = \left(\frac{\delta m_{\text{atm}}}{m_{\text{atm}}}(t)\right)$$

$$+ \left(\frac{\delta P_{\oplus}(t)}{P_{\oplus}(t)}\right) = \left(\frac{\delta m_{\text{atm}}}{m_{\text{atm}}}(t)\right)$$

$$+ \left(\frac{\delta P_{\oplus}(t)}{P_{\oplus}(t)}\right).$$

(30)

Surely, the scattered luminosity $L_{\text{sc}}(t)$ will have to strongly scale in direct proportion with the surface area of the Earth on which the incident luminosity is incident, i.e. $[L_{\text{sc}}(t) \propto R_{\oplus}^2(t)]$. From this relation $[L_{\text{sc}}(t) \propto R_{\oplus}^2(t)]$, it follows that:

$$\frac{\delta L_{\text{sc}}(t)}{L_{\text{sc}}(t)} = 2 \left(\frac{\delta R_{\oplus}(t)}{R_{\oplus}(t)}\right),$$

(31)

and hence:

$$\frac{\delta L_{\text{sc}}(t)}{L_{\text{sc}}(t)} = 2 \left(\frac{\delta m_{\text{atm}}}{m_{\text{atm}}}(t)\right) - \delta P_{\oplus}(t).$$

(32)

7 Changing Earth Albedo

In its simplest definition, the albedo ($A$) is the ratio of the incident luminosity [$L_{\text{in}}(t) = \pi R_{\text{eff}}^2(t) F_{\oplus}(r, t)$] to the scattered luminosity [$L_{\text{sc}}(t)$], i.e.:

$$A(t) = \frac{L_{\text{in}}(t)}{L_{\text{sc}}(t)}.$$

(29)

therefore — for the Earth-Sun system, a change in $A_{\oplus}(t)$ will depend on changes in $L_{\text{sc}}(t)$ and $L_{\text{in}}(t)$, i.e.:

$$\frac{\delta A_{\oplus}(t)}{A_{\oplus}(t)} = \frac{\delta L_{\text{sc}}(t)}{L_{\text{sc}}(t)} - \frac{\delta L_{\text{in}}(t)}{L_{\text{in}}(t)}.$$

(30)

Surely, the scattered luminosity $L_{\text{sc}}(t)$ will have to strongly scale in direct proportion with the surface area of the Earth on which the incident luminosity is incident, i.e. $[L_{\text{sc}}(t) \propto R_{\oplus}^2(t)]$. From this relation $[L_{\text{sc}}(t) \propto R_{\oplus}^2(t)]$, it follows that:

$$\frac{\delta L_{\text{sc}}(t)}{L_{\text{sc}}(t)} = 2 \left(\frac{\delta R_{\oplus}(t)}{R_{\oplus}(t)}\right),$$

(31)

and hence:

$$\frac{\delta A_{\oplus}(t)}{A_{\oplus}(t)} = 2 \left(\frac{\delta m_{\text{atm}}}{m_{\text{atm}}}(t)\right) - \frac{\delta P_{\oplus}(t)}{P_{\oplus}(t)}.$$

(32)
thus:

$$\frac{\delta A_\oplus(t)}{A_\oplus(t)} = 2 \left[ \frac{\delta R_\oplus(t)}{R_\oplus(t)} - \frac{\delta R_{\text{eff}}(t)}{R_{\text{eff}}(t)} \right] \frac{\delta L_\oplus(t)}{L_\oplus(t)} + 2 \frac{\delta r_\oplus(t)}{r_\oplus(t)}$$

(33)

Neglecting the term $\delta r_\oplus(t)/r_\oplus(t)$ [since according to equation (4), it is much smaller that $\delta L_\oplus/L_\oplus$] and using equation (27) to substitute for $\delta R_{\text{eff}}/R_{\text{eff}}$, we will have:

$$\frac{\delta A_\oplus(t)}{A_\oplus(t)} = \frac{2(1 - \gamma_\oplus^3)}{3} \frac{\delta R_\oplus(t)}{R_\oplus(t)} - \frac{\delta L_\oplus(t)}{L_\oplus(t)}.$$  

(34)

This equation (34) relates four quantities together with their changes, namely the albedo $A$, the Earth’s radius $R_\oplus$, the Solar luminosity $L_\oplus$ and the height of the Earth’s atmosphere via the $\gamma$-parameter. In the next section, we shall now link the Earth’s albedo to both the supposed expansion rate of the Earth and the changing luminosity of the Sun.

8 Earth as a Delicate Incubator

Life is delicate and requires steady and predictable conditions for it to flourish. In the case of a planet harbouring life like our Earth, one of these conditions is the sustenance of steady average global temperatures. While the effective surface temperature of the Earth $T_\oplus(t)$, or any planet for that matter, is not a good measure of the actual surface temperature — the steadiness of this temperature $T_\oplus(t)$ is to a good and reasonable degree of approximation, a good measure of the steadiness in the average global temperatures of the Earth (or any planet in question). If during the Archean eon life existed on Earth — and still exists today, it would make sense to entertain the idea that the Earth has maintained the requisite predictable steady conditions to sustain life from that period to the present day. As such, it makes sense to assume that the Earth has maintained the same average global temperatures throughout its life — in this way, the Earth must — one way or the other — be a very good incubator of some sort.

From the forgoing, steady average global temperatures imply steady average effective global surface temperature $T_{\oplus\text{(t)}}$, i.e., $[\delta T_\oplus(t) = 0]$. This condition $[\delta T_\oplus(t) = 0]$, when applied to equation (14), leads to:

$$\frac{\delta F_\oplus(r, t)}{F_\oplus(r, t)} = \frac{A_\oplus(t)}{1 - A_\oplus(t)} \frac{\delta A_\oplus(t)}{A_\oplus(t)}.$$  

(35)

Secular changes in the Earth’s planetary albedo have been proposed as one of the important parameters controlling climate on the early Earth by e.g. Rossow et al. (1982) and Charlson et al. (1987), so the evocation of this idea here is not a strange idea at all, but plausible one.

Now, equation (35) can be re-written as:

$$\frac{\delta L_\oplus(t)}{L_\oplus(t)} - \frac{2(1 - \gamma_\oplus^3)}{3} \frac{\delta A_\oplus(t)}{A_\oplus(t)} = \frac{A_\oplus(t)}{1 - A_\oplus(t)} \frac{\delta A_\oplus(t)}{A_\oplus(t)}.$$  

(36)

As before [i.e., at the instance of equation (34)], neglecting the term $\delta r_\oplus(t)/r_\oplus(t)$ in equation (36) [since according to equation (4), it is much smaller that $\delta L_\oplus/L_\oplus$], it follows that:

$$\frac{\delta L_\oplus(t)}{L_\oplus(t)} \simeq \frac{A_\oplus(t)}{1 - A_\oplus(t)} \frac{\delta A_\oplus(t)}{A_\oplus(t)}.$$  

(37)

Substituting $\delta A_\oplus(t)/A_\oplus(t)$ from equation (34) into equation (37) and thereafter rearranging, one obtains:

$$\frac{\delta R_\oplus(t)}{R_\oplus(t)} \simeq -\frac{3}{2A_\oplus(t)(1 - \gamma_\oplus^3)} \frac{\delta L_\oplus(t)}{L_\oplus(t)}.$$  

(38)

Equation (38), connects the Earth’s expansion rate and the Solar luminosity rate. It is important at this point for one to notice that for an expanding Earth [i.e., $\delta R_\oplus(t) > 0$], we need to have $[\delta L_\oplus(t) < 0]$, since for all values of $\gamma$, we have that $[1 - \gamma^3 > 0]$. It is for this reason that we had to re-consider the sign of $\delta L_\oplus(t)$ at the instance of equation (4). Now, after all the preparation, in the next section, we now present our suggested solution to the FYS-Paradox, where the Earth system is cast as an automatic self-regulating incubator.

9 Suggested Solution

What equation (38) implies is that, if the Earth where a delicate incubator of life as supposed in the previous section — i.e., a delicate incubator that maintains steady average global temperatures via steady average effective global surface temperature $T_{\oplus\text{(t)}}$ defined in equation (14), i.e., $[\delta T_\oplus(t) = 0]$, then, the Earth can do so for a steadily changing Solar luminosity by responding to this change in Solar luminosity via global radial expansion (or contraction) of the Earth. Actually, it would appear as though the expansion of the Earth is a means of auto-self-regulating the mean global temperatures.

Now, — first, using the Kármán line, at $\sim 100 \text{ km}$, [i.e., $H_\oplus(t_0) = 0.0157 R_\oplus(t_0)$] as the height of the Earth’s atmosphere, we shall now insert the numbers into equation (38). Let $t_0$ represent the present cosmic epoch. The present day albedo of the Earth is $\sim 0.29$, i.e.: $[A_\oplus(t_0) = 0.29]$ (e.g., Stephens et al. 2015). The mean radius of the solid Earth is $\sim 6.40 \times 10^6 \text{ m}$, i.e.: $[R_\oplus(t_0) = 6.40 \times 10^6 \text{ m}]$, hence
\[ \gamma_{\oplus}(t_0) = 1/64 \]. From all this, it follows that from the *Archaean eon* to the present epoch \( t_0 \), we must have:

\[ \frac{\delta R_{\oplus}(t_0)}{R_{\oplus}(t_0)} = +1.29. \tag{39} \]

Equation (39) corresponds to a present day Earth radial expansion of \( \sim 2.60 \pm 0.60 \text{ mm/yr} \) and is about seven times the expansion rate measured by Shen et al. (2015). On the one hand, we can drop the *Kármán line* is the boundary of of the Earth’s atmosphere and ask, what is the height of the Earth’s atmosphere that would correspond to Shen et al. (2015)’s Earth expansion rate of \( \sim 0.36 \pm 0.06 \text{ mm/yr} \)? For this, we make \( \gamma \) the subject of the formula in equation (38), i.e.:

\[ \frac{H_{\oplus}(t)}{R_{\oplus}(t)} = 1 - \left[ 1 + 3 \frac{A_{\oplus}(t)}{2 \cdot \delta L_{\oplus}(t)/\delta R_{\oplus}(t)} \right]^2 \frac{1}{3}. \tag{40} \]

Inserting the numbers for these quantities for the present epoch, one obtains that:

\[ \frac{H_{\oplus}(t_0)}{R_{\oplus}(t_0)} = 1.54 \pm 0.05. \tag{41} \]

What equation (41) is telling is that if the present day Earth’s atmosphere has a height of \( \sim 9860 \text{ km} \), the Earth’s average global temperatures have been steady since the *Archaean eon*. Equation (40) can be written in-terms of \( \dot{L}_{\oplus}(t) \) and \( \dot{R}_{\oplus}(t) \) instead of \( \delta L_{\oplus}(t) \) and \( \delta R_{\oplus}(t) \) and in this way, it allows one to evaluate \( H_{\oplus}(t)/R_{\oplus}(t) \) for any given epoch, i.e.:

\[ \frac{H_{\oplus}(t)}{R_{\oplus}(t)} = 1 - \left[ 1 + 3 \frac{A_{\oplus}(t)}{2 \cdot \dot{L}_{\oplus}(t)/\dot{R}_{\oplus}(t)} \right]^2 \frac{1}{3}. \tag{42} \]

In the next section, we shall discuss the results of the present section as well as give an overall general discussion.

### 10 General Discussion

Despite our strong belief (as expressed in the readings, Nyambuya 2014a,b) in the *Expanding Earth Hypothesis*, this reading has been presented in a manner that does not advocate (vouch) for either position of expansion or no-expansion. We have merely argued that the idea of an expanding Earth has a place — perhaps, an important one — in the ‘convoluted matrix’ of possible solutions to this long standing paleoclimatology riddle of the FYS-Paradox. If anything, our suggestion is new insofar as solutions to this problem is concerned. At least in the wider literature that we have had the fortune to lay our hands, nowhere does one come across a solution to this problem that makes use of an Expanding Earth Hypothesis.

The proposed model presents the expanding Earth as an auto self-regulating incubator which maintains steady average global temperatures by self-adjusting the boundary of the atmosphere. If the Earth is expanding as observations appear to indicate (Shen et al. 2015,b, 2011; Xu and Sun 2014; Wu et al. 2011; Chen 2000; Gerasimenko 1996, 1997, 1993), then, the boundary of the Earth’s atmosphere must also change in response to this solid Earth expansion. At present, the boundary of the Earth’s atmosphere is not known with any exactitude thus making it difficult to check the proposed model. The *Kármán line* as the boundary of the Earth’s atmosphere requires a solid Earth expansion rate of \( \sim 2.60 \pm 0.60 \text{ mm/yr} \) and this is about seven times the latest Earth expansion rate measured by Shen et al. (2015), thus making this boundary not favorable insofar as the proposed model and observations is concerned. If we hold the model to be true, then — according to the proposed model, Shen et al. (2015)’s Earth’s expansion of \( \sim 0.36 \pm 0.06 \text{ mm/yr} \) requires an Earth’s atmospheric height of \( \sim 9860 \text{ km} \). Invariably, what this would mean is that in the region \([100 \text{ km} < H_{\oplus}(t_0) \leq 9860 \text{ km}] \), there must exist a very thin atmosphere that should be difficult to detect.

The question is: *Can the Earth sustain an atmosphere in the region* \([100 \text{ km} < H_{\oplus}(t_0) \leq 9860 \text{ km}] \)? The answer is yes. In order for the Earth to have an atmosphere in the region \([100 \text{ km} < H_{\oplus}(t_0) \leq 9860 \text{ km}] \) all there is to it is whether or not the molecules in this region have enough thermal energy to escape the gravitational influence of the Earth — the thermal energy \( 3k_B T_{bg}(t)/2 \) for the molecule at the very edge of the Earth’s atmosphere must be less than the gravitational potential energy at the surface \([r = R_{\text{eff}}(t)] \), i.e.:

\[ \frac{3}{2} k_B T_{bg}(t) \leq \frac{\mu_{\oplus} G (M_{\oplus} + m_{\text{atm}}) m_H}{R_{\text{eff}}(t)}, \tag{43} \]

where \( \mu_{\oplus} \) mean number of molecules per unit of the Earth’s atmosphere. From equation (43), it follows that:

\[ T_{bg}(t_0) \leq \frac{2 \mu_{\oplus} G (M_{\oplus} + m_{\text{atm}}) m_H}{3k_B R_{\text{eff}}(t)} \tag{44} \]

In order to compute \( T_{bg}(t_0) \), we need to know mean number of molecular \( \mu_{\oplus} \) for a one average unit of the Earth’s atmosphere and this we need to know the Earth’s atmospheric composition. Table 2 lists the composition of the of Earth’s atmosphere by volume and by mass. In general, it (Earth’s atmosphere) comprises nine major constituents and these
are: Nitrogen, Oxygen, Argon, Carbon dioxide, Neon, Helium, Methane, Krypton and Hydrogen. Accordingly, $\mu_{\oplus}$:

$$\mu_{\oplus} = \frac{1}{\text{amu}} \left( \sum_{i=1}^{9} w_i A_i \right),$$  \hspace{1cm} (45)

where $w_i$ is the percentage of constituent ‘$i$’ by mass and $A_i$ the atomic weight of this constituent: amu is the usual atomic mass unit which equals 1.66053904(20) $\times 10^{-27}$ kg. From equation (45) and the values given in Table 2, it follows that ($\mu_{\oplus} = 28.80$), hence, substituting ($\mu_{\oplus} = 28.80$) into equation (44), we obtain that $[T_{ bg}(t_{0}) \leq 46400 \text{ K}]$. The Earth itself is much cooler than the required 46400 K for it to maintain an atmosphere at the distance of $1.54R_{\oplus}$; the meaning of which is that yes, the Earth may very well have a very thin atmosphere in the supposed region [100 km < $H_{\oplus}(t_{0}) \leq 9860 \text{ km}$]. That is to say, from a thermodynamic and gravitational view point, there absolutely is nothing to stop the Earth from having an atmosphere in the region [100 km < $H_{\oplus}(t_{0}) \leq 9860 \text{ km}$].

10.1 Life and Conditions for the Early Earth

The (herein) proposed model of the Earth and its atmosphere has some interesting outcomes when it comes to the early Earth’s surface pressure and gravitational acceleration. For a non-accreting Earth with steady global average temperatures, it follows from equation (24), that:

$$\frac{\delta P_{\oplus}(t)}{P_{\oplus}(t)} = \frac{\delta g_{\oplus}(t)}{g_{\oplus}(t)} = -3 \left( \frac{\delta R_{\oplus}(t)}{R_{\oplus}(t)} \right).$$  \hspace{1cm} (46)

This equation (46) informs us that for an expanding Earth, we will have a three-fold decrease in both the rate of change of the density of the Earth’s atmosphere and the surface pressure. For example, if according to equation (39) the Earth’s radius has increased by 130% over the past 3.20 billion years, then, the surface pressure and the mean density of the atmosphere back then should have been 390% the present day surface pressure and density of the atmosphere. For the pressure, this means a surface pressure of i.e. 4.90 atm. Such conditions of high surface pressure and atmospheric density would have required ‘giant animals’ that are significantly different from present day life.

For the surface gravity of the Earth, we have that:

$$\frac{\delta g_{\oplus}(t)}{g_{\oplus}(t)} = -2 \left( \frac{\delta R_{\oplus}(t)}{R_{\oplus}(t)} \right).$$  \hspace{1cm} (47)

Thus for $[\delta R_{\oplus}(t_{0})/R_{\oplus}(t_{0}) \sim +1.29]$ over the past 3.20 billion years, it follows from equation (47), that the gravitational field intensity $[g_{\oplus}(t_{A})]$ then, was about 3.58 the present day gravitational field intensity $[g_{\oplus}(t_{0}) \sim 9.80 \text{ m/s}^2]$, i.e., $[g_{\oplus}(t_{A}) \sim 35.08 \text{ m/s}^2]$. A larger gravitational field intensity further supports the idea of ‘giant animals’ in the Early Earth because the bone structure of any animal in a higher gravitational field intensity will have to be much stronger in-order to stand the stress induced by this higher gravitational field intensity. It is only logical that way.

In terms of the body size of dinosaurs, the above findings — of atmospheric pressure, density and surface gravity — are interesting. Convincing archaeological evidence informs us that dinosaurs once roamed the Earth and these creatures where humongous insofar as their body size is concerned. As to why they where that big, no body really knows for sure. In light of foregoing, if the surface gravity, atmospheric pressure and density of the Earth’s where significantly larger than today, clearly, any animal that pumps blood around its body in-order to sustain life would have required a much bigger in-order to stand these extreme conditions. Surely, conditions for the early Earth would have required life to be significantly different from the life we see today. As the conditions of pressure and density changed gradually, so should have life on Earth. This is the essence and central tenant of Darwinian evolution.

10.2 Conditions for the Existence of Exo-Life

Exo-Life is life on an exoplanet where an exoplanet is a planet outside our own Solar system that orbits a star. As of April 2018, there are more than 3700 confirmed exoplanets in at least 612 exoplanetary systems. The conditions of the Early Earth’s surface gravity, atmospheric pressure and gravitational field intensity prevailing during the period $[t_{A} \leq t \leq t_{0}]$ can be used to probe exo-life.

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Table 2  Composition of the Earth’s Dry Atmosphere (cf., Hartmann et al. 2013).

<table>
<thead>
<tr>
<th>Gas</th>
<th>Volume (%)</th>
<th>Mass (%)</th>
<th>Atomic Mass (amu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>78.084000</td>
<td>75.523000</td>
<td>28</td>
</tr>
<tr>
<td>Oxygen</td>
<td>20.94000</td>
<td>23.133000</td>
<td>32</td>
</tr>
<tr>
<td>Argon</td>
<td>0.934000</td>
<td>1.288000</td>
<td>18</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.038100</td>
<td>0.053000</td>
<td>44</td>
</tr>
<tr>
<td>Neon</td>
<td>0.001818</td>
<td>0.001267</td>
<td>10</td>
</tr>
<tr>
<td>Helium</td>
<td>0.000524</td>
<td>0.001267</td>
<td>2</td>
</tr>
<tr>
<td>Methane</td>
<td>0.000175</td>
<td>0.000290</td>
<td>16</td>
</tr>
<tr>
<td>Krypton</td>
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<td>0.000724</td>
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</tr>
<tr>
<td>Hydrogen</td>
<td>0.000055</td>
<td>0.000038</td>
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</tr>
</tbody>
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7See e.g. https://exoplanetarchive.ipac.caltech.edu/. Visited on this day Sunday, 1 April, 2018@22h38 GMT+2.
10.3 Conclusion

1. *In-principle*, the Expanding Earth Hypothesis can explain the so-called Faint Young Sun Paradox *via* an automatic self-regulating mechanism where the height of the Earth’s atmosphere re-adjusts and in-turn the albedo changes in such a manner that it maintains constant average global temperatures for so long as the Earth is expanding and the Sun is getting brighter and brighter with time.

2. In-accordance with the proposed expanding Earth evolutionary model, Shen et al. (2015, 2011)’s measurements of an Earth expansion of $\sim +0.36(6)$ mm/yr require that the height of the Earth’s atmosphere be $\sim 9860$ km, the invariable meaning of which is that in the supposed region $[100 \text{ km} < H_\oplus(t_0) \leq 9860 \text{ km}]$, there must exist a very thin atmosphere which has so far escaped detection, since space rockets have not detected any atmosphere in this region *i.e.*, in the region beyond $\sim 100$ km.
References


URL http://acmg.seas.harvard.edu/publications/


URL http://online.kitp.ucsb.edu/online/exoplanets c10/kasting/pdf/Kasting_ExoPlanetsConf_KITP.pdf


