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Modelling the effects of heavy alcohol consumption on the transmission dynamics of gonorrhea

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Abstract Prior studies have indicated that heavy alcohol drinkers are likely to engage in risky sexual behaviours and thus, more likely to get sexually transmitted infections (STIs) than social drinkers. Here, we formulate a deterministic model for evaluating the impact of heavy alcohol drinking on the reemerging gonorrhea epidemic. The model is rigorously analysed, showing the existence of a globally asymptotically stable disease-free equilibrium whenever the reproductive number is less than unity. If the disease threshold number is greater than unity, a unique endemic equilibrium exists and is globally asymptotically stable in the interior of the feasible region and the disease persists at endemic proportions if it is initially present. Both analytical and numerical results are provided to ascertain whether heavy alcohol drinking has an impact on the transmission dynamics of gonorrhea.

Keywords Gonorrhea · Alcohol · Reproductive number · Stability

1 Introduction

Gonorrhea is a very common infectious disease, which was linked to a sharp decline in the early 1980s, [8, 19, 25] in coincidence with reports of reductions in unsafe sexual behaviours [26, 27]. However, several recent reports suggest a reversal in these trends. In the year 1999, investigators in Seattle, USA reported increases in syphilis, gonorrhea, and chlamydia infection [9, 44]. Sexually transmitted infections (STIs) other than HIV account for a significant portion of illness worldwide, with more than 340 million new cases of curable STIs (mainly gonorrhea, syphilis, chlamydia and trichomoniasis) occurring globally in adults aged 15 to 49 each year [45]. Together, HIV and sexually transmitted infections (STIs) are responsible for the destruction of health on a massive scale worldwide [32]. An estimated 39.5 million people live with HIV worldwide, of these 4.3 million were infected in 2006 alone and 24.7 million live in sub-Saharan Africa, the region of the world currently experiencing the highest concentration of global emergencies [41]. In addition, STIs facilitate the transmission of HIV infection [45]. Cross-sectional studies in sub-Saharan Africa have demonstrated that heavy alcohol drinking is associated with risky sexual behaviours [18, 31, 39], sexually transmitted disease prevalence [29, 39]. Sex
under the influence of alcohol is associated with both increased STIs prevalence and a greater likelihood of paying for sex [18, 30, 39]. Heavy alcohol drinking is also associated with a greater likelihood of condom failure and improper use of lubricants with condoms [39, 40].

Gonorrhea is a sexually transmitted disease caused by the bacterium called *Neisseria gonorrhoeae*. It is passed from one person to another during vaginal, anal, and oral sex. It can be found in the throat, vagina, urethra, and anus. Babies can be infected during birth, causing eye infections. Gonorrhea symptoms appear within 10 days after a person is exposed to the germ, then they disappear. Women often have no symptoms at all, but both women and men whose symptoms have disappeared are still infected. Those infected in the throat through oral sex may feel like having a sore throat. Gonorrhea is more easily spread to the throat by penis–mouth sex and rarely by mouth–vagina sex.

In men, gonorrhea can cause epididymitis, a painful condition of the testicles that can sometimes lead to infertility if left untreated. Without prompt treatment, gonorrhea can also affect the prostate and can lead to scarring inside the urethra, making urination difficult. The disease can spread to the blood or joints [11, 21, 32], a condition that can be life-threatening. Also, individuals with gonorrhea can more easily contract HIV, the virus that causes AIDS. Individuals with HIV infection and gonorrhea are more likely to transmit HIV to someone else than persons with HIV infection alone [11, 32]. The predominant transmission mode of both HIV and other STIs is sexual intercourse. Methods for preventing sexual transmission of HIV and STIs are the same, as are the target audiences for interventions. High rates of STIs continue to be experienced in sub-Saharan African (especially the Southern African Region), and this is of great concern since STIs facilitate the transmission of HIV [38]. HIV and STIs spread and kill most quickly in populations affected by poverty, social unrest and lack of health infrastructure. These factors are commonly present in humanitarian emergencies [32].

Although, there are various factors which play significant role on the spread of gonorrhea epidemic in the community such as social, cultural, economic and so on, but as of late various studies across the world [13, 18] are increasingly lending support and evidence to heavy alcohol drinking as one of the root causes for the rapid spread of sexually transmitted infections. In this study, the term heavy alcohol drinking refers to male individuals who consume more than 14 drinks per week or more than five drinks per occasion and women who drink more than seven drinks per week or more than four drinks per occasion; individuals who consume alcohol below the above defined threshold will be classified as non-drinkers or light drinkers; we refer to such drinking as normal alcohol consumption [10, 15, 33, 42]. In a cross-cultural study conducted by WHO [46] in eight countries, namely Kenya, South Africa, Zambia, Belarus, Romania, Russian Federation, India and Mexico, it was found that alcohol intake correlated positively with several risky sex indicators like regretted intercourse, number of sex-partners, etc. The tax hike on beer in California in 1991 resulted in gonorrhea rates dropping by 30% the following year [40], and from this observation we can infer that heavy alcohol drinking is correlated to gonorrhea epidemic.

A brief survey on previous works provides the context of this paper. Various theoretical studies have been carried out on the mathematical modelling of gonorrhea transmission dynamics focusing on a number of different issues, see [5, 6, 14, 16, 20, 24, 32, 34, 37]. In 1996, Chavez et al. [4] investigated competitive exclusion in gonorrhea models and other STIs. In their work, the authors concluded that in a behaviourally and genetically homogeneous population co-existence is not possible except under very special circumstances. None of these studies has considered the effect of alcohol on the transmission dynamics of gonorrhea. It is therefore against this background that our study finds its relevance and motivation, by formulating a mathematical model to investigate the impact of heavy alcohol consumption on the transmission dynamics of gonorrhea epidemic in the community. The model incorporates some key epidemiological features such as recovery of treated gonorrhea infectives. The main objective in this study is to forecast future trends in the incidence of gonorrhea epidemic and also to quantify the association between heavy alcohol consumption and gonorrhea epidemic in the community.

The paper is structured as follows. The gonorrhea transmission model is formulated and simplified in the next section and the long-term dynamics are presented in Sect. 3. Simulation results and projection profiles of gonorrhea are presented in Sect. 4. Summary and concluding remarks round up the paper.
2 Model description

The total sexually-active population at time $t$, denoted by $N_{ND}$, is sub-divided into mutually-exclusive compartments, namely susceptibles non-heavy alcohol drinkers $S_N$, susceptibles heavy alcohol drinkers $S_D$, individuals infected with gonorrhea who are non-heavy alcohol drinkers $I_N$, and individuals infected with gonorrhea who are heavy alcohol drinkers $I_D$. Then, $N_{ND} = S_N + S_D + I_N + I_D$. The susceptible population is increased by a constant inflow into the population at rate $\Lambda$. A fraction $\pi_0$ of these individuals are assumed to be non-heavy alcohol drinkers and the complementary fraction, $\pi_1 = 1 - \pi_0$ are heavy alcohol drinkers. This is reasonable because the underlying population is made of adults only and juveniles mature into these two classes (influence from some peers is most probably the main driver behind juvenile drinking and hence their recruitment into the heavy alcohol drinkers class). Susceptible individuals acquire infection following contact with gonorrhea infectives at a rate $\lambda$. Assuming homogeneous mixing of population, the susceptible individuals acquire gonorrhea infection at a rate $\lambda_{ND}$, where $\lambda_{ND} = \frac{\beta(I_N + \eta I_D)}{N_{ND}}$, with the parameter $\beta$ denoting the effective contact rate for gonorrhea transmission. The modification parameter $\eta > 1$ captures the fact that gonorrhea infectives who are heavy alcohol drinkers $I_D$ are assumed to have a higher chance, relative to gonorrhea infectives who are non-heavy alcohol drinkers $I_N$ of generating new infections due to their risky sexual behaviours [35]. It is assumed that susceptible individuals who are heavy alcohol drinkers $S_D$ have a higher chance, in relation to susceptibles who are non-alcohol heavy drinkers $S_N$ of acquiring gonorrhea infections following contact with the respective infected class by a factor $\sigma \geq 1$ [35]. Upon receiving treatment, gonorrhea infectives are assumed to recover and return to the susceptible class at a rate $\phi$. Furthermore, natural mortality occurs in all classes at a constant rate $\mu$. Individuals are assumed to either become heavy alcohol drinkers maybe by choice, peer pressure or any other reasons at rate $\alpha$, and likewise individuals may cease to be heavy alcohol drinkers (possibly due to counselling, health reasons, pressure at work, poverty or any other reason) at a rate $\gamma$. This study makes an assumption that the population is uniform and homogeneously mixing, although we note that models where transmission is restricted to occur through sexual partnerships give different predictions [2, 28]. The model flow diagram is depicted in Fig. 1.

The aforementioned assumptions and description above give rise to the following system of ordinary differential equations

\begin{align}
S_N' &= \pi_0 \Lambda + \phi I_N + \gamma S_D - \lambda S_N - (\mu + \alpha) S_N,
S_D' &= \pi_1 \Lambda + \phi I_D + \alpha S_N - \sigma \lambda S_D - (\mu + \gamma) S_D,
I_N' &= \lambda S_N + \gamma I_D - (\mu + \phi + \alpha) I_N,
I_D' &= \sigma \lambda S_D + \alpha I_N - (\mu + \phi + \gamma) I_D.
\end{align}

(1)

2.1 Model basic properties

In this section, we study the basic results of solutions of model system (1), which are essential in the proofs of stability results.

**Lemma 1** The nonnegative orthant $\mathbb{R}_+^4$ is positively invariant for the system (1).

**Proof** Model system (1) can be written in the form $X' = AX + B$, with

![Fig. 1 Model flow diagram](image-url)
Note that $B \geq 0$ and $A$ is a Metzler matrix ($A$ Metzler matrix is a matrix with off-diagonal entries non-negative [3], which implies that the system (1) is positively invariant in $\mathbb{R}_+^4$. □

**Lemma 2** Each non-negative solution is bounded in $L^1$-norm by $\max\{N(0), \Lambda/\mu\}$.

*Proof* The $L^1$ norm of each non-negative solution is $N$ and it satisfies the inequality $N' \leq \Lambda - \mu N$. Solutions to the equation $M' = \Lambda - \mu M$ are monotone increasing and bounded by $\Lambda/\mu$ if $M(0) < \Lambda/\mu$. They are monotone decreasing and bounded above if $M(0) \geq \Lambda/\mu$. Since $N' \leq M'$ the claim follows. □

**Corollary 1** The region

$$
\Phi = \left\{(S_N, S_D, I_N, I_D) \in \mathbb{R}_+^4 : 
\left\{N_{ND} \leq \max\left\{N(0), \frac{\Lambda}{\mu}\right\}\right\}\right. ,
$$

(3)

is invariant and attracting for system (1).

Thus, the model is mathematically and epidemiologically well-posed and it is sufficient to consider the dynamics of the flow generated by system (1) in $\Phi$.

**Theorem 1** For every non-zero, non-negative initial value, solutions of model system (1) exist for all times.

*Proof* Local existence of solutions follows from standard arguments since the right-hand side of (1) is locally Lipschitz continuous. Global existence follows from the *a priori* bounds. □

The model has a number of invariant sets that correspond to epidemiologically limiting cases of the problem. The effects of heavy alcohol drinking on the spread of gonorrhea epidemic is modelled via the additional infection pathways with corresponding parameters $\eta$ and $\sigma$, with the recruitment of individuals into the two classes determined by $\pi_0, \pi_1$. If we set $\eta = 1$ and $\sigma = 1$ then we can simply add the respective compartments $S = S_N + S_D$ and so on, and we obtain a standard SIS-model. If, on the other hand, we allow only one group to enter the system then we obtain invariant sets.

**Lemma 3** If $\pi_0 = 0$ then the set $\{S_N = I_N = 0\}$ is invariant and attracting for system (1). If $\pi_1 = 0$ then the set $\{S_D = I_D = 0\}$ is invariant and attracting for system (1).

### 3 Analytical results

In this section, we derive the equilibrium states, disease-free equilibrium (DFE) and endemic equilibrium (EE), and investigate their stability using the basic reproductive number.

#### 3.1 Disease-free equilibrium and the basic reproductive number

Model system (1) has an evident DFE given by

$$
E^0 = (S_N, S_D, I_N, I_D) = \left(\frac{\Lambda(\mu \pi_0 + \gamma)}{\mu(\alpha + \gamma + \mu)}, \frac{\Lambda(\mu \pi_1 + \alpha)}{\mu(\alpha + \gamma + \mu)}, 0, 0\right) ,
$$

(4)

The linear stability of $E^0$ is governed by the basic reproductive number which is defined as the spectral radius of the next generation matrix [43]. The basic reproduction number is the expected number of secondary infections generated by a single infectious individual during his/her entire infectious period. However, in our study an infectious individual can either be in $I_N$ or in $I_D$ and the expected number of secondary infections depends on the class. We consider the different possibilities in detail.
3.1.1 Case 1: There are no heavy alcohol drinkers in the community

In this case, we set \( \pi_1 = \alpha = \gamma = S_D = I_D = 0 \). Following van den Driessche and Watmough [43], the reproductive number is given by

\[
R_N = \frac{\beta}{\mu + \phi}.
\] (5)

This reproductive rate sometimes referred to as the back of the napkin [1] is simply the ratio of the per capita rate of infection and the average lifetime of an individual in class \( IN \). It is defined as the number of secondary gonorrhea cases produced by a single infected individual during his/her entire infectious period in a totally naive (susceptible) population in the absence of heavy alcohol drinkers.

3.1.2 Case 2: The entire population is made up of heavy alcohol drinkers

In this case, we set \( \pi_0 = \alpha = \gamma = S_N = I_N = 0 \). Again following the approach in [43], the reproductive number is given by

\[
R_D = \frac{\beta \sigma \eta}{\mu + \phi}.
\] (6)

\( R_D \) measures the average number of new infections generated by a single gonorrhea infective who is also a heavy alcohol drinker during his/her entire infectious period when he/she is introduced into a susceptible population of heavy alcohol drinkers.

3.1.3 Case 3: Co-existence of non-alcohol heavy drinkers and heavy alcohol drinkers in the community

When \( \pi_0, \pi_1 > 0 \), following van den Driessche and Watmough [43] and using the notation defined therein, the matrices \( F \) and \( V \) for the new infection terms and the remaining transfer terms are respectively given by

\[
F = \begin{bmatrix}
\frac{\beta(\gamma + \mu \pi_0)}{\alpha + \gamma + \mu} & \frac{\beta \eta}{\alpha + \gamma + \mu} \\
\frac{\beta \eta}{\alpha + \gamma + \mu} & \frac{\beta \eta}{\alpha + \gamma + \mu}
\end{bmatrix}
\] and

\[
V = \begin{bmatrix}
\alpha + \mu + \phi & -\gamma \\
-\alpha & \gamma + \mu + \phi
\end{bmatrix}.
\] (7)

It follows that the reproduction number for model system (1), denoted by \( R_{ND} \), is given by

\[
R_{ND} = \frac{\beta [\sigma (\eta k_1 + \gamma)(\alpha + \mu \pi_1) + (k_2 + \alpha \eta)(\gamma + \mu \pi_0)]}{k_3(k_1 k_2 - \alpha \gamma)},
\]

where \( k_1 = \alpha + \mu + \phi, \ k_2 = \gamma + \mu + \phi, \ k_3 = \alpha + \gamma + \mu, \) and \( k_1 k_2 > \alpha \gamma \). From (5) and (6), \( \sigma = \frac{R_D}{R_N} \) so that

\[
R_{ND} = \frac{\beta [\sigma (\eta k_1 + \gamma)(\alpha + \mu \pi_1) + R_D \eta (k_2 + \alpha \eta)(\gamma + \mu \pi_0)]}{R_N \eta k_3(k_1 k_2 - \alpha \gamma)}.
\] (8)

Using Theorem 2 in [43], the following result is established.

**Theorem 2** The DFE \((E^0)\) of model system (1) is locally-asymptotically stable (LAS) if \( R_{ND} < 1 \), and unstable if \( R_{ND} > 1 \).

The reproductive number \( R_{ND} \) measures the average number of new infections generated by a single infected individual in a completely susceptible population. Thus, Theorem 2 implies that gonorrhea can be eliminated from the community (whenever \( R_{ND} < 1 \)) if the initial sizes of the sub-populations of the model system (1) are in the basin of attraction of the DFE \((E^0)\). To ensure that elimination of gonorrhea is independent of the initial sizes of the sub-populations, it is necessary to show that the DFE is globally stable.

Using a theorem from Castillo-Chavez et al. [7], we now show the global stability of the DFE in the case that the basic reproduction number is less than unity \((R_{ND} < 1)\).

**Theorem 3** The DFE \( E^0 \) of our model system (1) is globally asymptotically stable provided \( R_{ND} < 1 \), and unstable if \( R_{ND} > 1 \).

**Proof** Following Castillo-Chavez et al. [7], we write system (1) in the form

\[
X'(t) = F(X, Y),
\]

\[
Y'(t) = G(X, Y), \quad G(X, 0) = 0,
\] (9)

where \( X = (S_N, S_D) \) denote the number of uninfected individuals, and the components of \( Y \in \mathbb{R}_+^2 \) denote the number of infected individuals. The disease-free equilibrium is now denoted by \( E^0 = (X_0, 0) \) where
Adding item (i) and (ii) one gets

\[ X = \begin{cases} 0 = \frac{\lambda (\mu \pi_0 + \gamma)}{\mu (\alpha + \gamma + \mu)} & \text{if} \quad \mu, \alpha, \gamma > 0 \end{cases} \].

We have to prove that the two conditions

(H1) For \( X'(t) = F(X, 0) \), \( X \) is a globally asymptotically stable;

(H2) \( G(X, Y) = UY - \hat{G}(X, Y), \hat{G}(X, Y) \geq 0 \)

for \((X, Y) \in \Phi\)

are satisfied where \( \Phi_1 \) is a positively invariant attracting domain. Consider

\[
F(X, 0) = \begin{bmatrix} \Lambda \pi_0 + \gamma S_D - (\mu + \alpha)S_N \\ \Lambda \pi_1 + \alpha S_N - (\mu + \gamma)S_D \end{bmatrix},
\]

\[
U = \begin{bmatrix} \beta_1 (\gamma + \pi_0 - (\alpha + \phi + \mu) \alpha + \pi_1 + (\alpha + \gamma + \mu) \alpha + \gamma \\ \beta_2 (\alpha + \pi_1 + (\alpha + \gamma + \mu) \alpha + \gamma) \alpha + \gamma \end{bmatrix}.
\]

Thus,

\[
\hat{G}(X, Y) = \begin{bmatrix} \hat{G}_1(X, Y) \\ \hat{G}_2(X, Y) \end{bmatrix} = \begin{bmatrix} \beta_1 (I_N + \eta I_D) \left( \frac{\mu \pi_0 + \gamma}{\mu + \alpha + \gamma} - \frac{S_N}{N_{ND}} \right) \\ \sigma \beta_2 (I_N + \eta I_D) \left( \frac{\mu \pi_1 + \alpha}{\mu + \alpha + \gamma} - \frac{S_D}{N_{ND}} \right) \end{bmatrix}.
\]

Now we have to show that \( \hat{G}_1(X, Y) \) and \( \hat{G}_2(X, Y) \) are both positive. To do this, we prove by contradiction. Assume that statements in (12) are true:

\[
(i) \quad \frac{\mu \pi_0 + \gamma}{\mu + \alpha + \gamma} < \frac{S_N}{N_{ND}} \quad \text{and}
\]

\[
(ii) \quad \frac{\mu \pi_1 + \alpha}{\mu + \alpha + \gamma} < \frac{S_D}{N_{ND}}.
\]

Adding together inequalities (i) and (ii) in (12), we have that

\[
\frac{\mu (\pi_0 + \pi_1) + \alpha + \gamma}{\mu + \alpha + \gamma} < \frac{S_N + S_D}{N_{ND}} \Rightarrow 1 < 1,
\]

which is not true, i.e., a contradiction (since a number cannot be less than itself).

Equation (12) and equation (14) should justify that \( \hat{G}(X; Y) \geq 0 \). In order to prove this we shall use the direct proof, for example if

(i) \( X \geq 2 \),

(ii) \( Y \geq 1 \).

Adding item (i) and (ii) one gets \( X + Y \geq 3 \).

Using the above argument \( \hat{G}(X; Y) \geq 0 \) if and only if

\[
(i) \quad \frac{\mu \pi_0 + \gamma}{\mu + \alpha + \gamma} \geq \frac{S_N}{N_{ND}} \quad \text{and}
\]

\[
(ii) \quad \frac{\mu \pi_1 + \alpha}{\mu + \alpha + \gamma} \geq \frac{S_D}{N_{ND}}.
\]

Adding item (i) and (ii) one gets

\[
\frac{\mu (\pi_0 + \pi_1) + \alpha + \gamma}{\mu + \alpha + \gamma} \geq \frac{S_N + S_D}{N_{ND}} \Rightarrow 1 \geq 1.
\]

Thus, from the above argument \( \hat{G}(X, Y) \geq 0 \). Therefore, the disease-free equilibrium \( e^0 \) is globally asymptotically stable. 

\[ \square \]

3.2 Endemic equilibria and stability analysis

Model system (1) has three possible endemic equilibria: the alcohol-free endemic equilibrium with a population of non-heavy alcohol drinkers only, the endemic equilibrium when the whole population is made up of heavy alcohol drinkers and the equilibrium where non-heavy alcohol drinkers and heavy alcohol drinkers co-exist, herein referred to as the interior equilibrium point.

3.2.1 Absence of heavy alcohol drinkers endemic equilibrium

We set \( \pi_0 = 1, \pi_1 = \alpha = \gamma = 0 \), so that there are no heavy alcohol drinkers in the community. Thus, system (1) reduces to

\[
S_N = \Lambda + \phi I_N^s - \frac{\beta I_N^s S_N^s}{N_N^s} - \mu S_N^s,
\]

\[
I_N^s = \beta I_N^s S_N^s \left( \frac{N_N^s}{N_N^s} \right) - (\phi + \mu)I_N^s.
\]

Summing the two equations in system (15), one gets

\[ N_N^s = \Lambda / \mu \]. Furthermore, from the second equation of system (15) \( I_N^s = 0 \) or
\[ \beta S_N^* = (\phi + \mu)N_N^* \Rightarrow S_N^* = \frac{(\phi + \mu)N_N^*}{\beta} \]
\[ \Rightarrow S_N^* = \frac{\Lambda}{\mu \mathcal{R}_N}, \quad (16) \]

with \( \mathcal{R}_N \) as defined in (5).

It follows from \( N_N^* = S_N^* + I_N^* \) that \( I_N^* = N_N^* - S_N^* \), thus
\[ I_N^* = \frac{\Lambda}{\mu} \left( 1 - \frac{1}{\mathcal{R}_N} \right) = (\mathcal{R}_N - 1)S_N^*. \quad (17) \]

Thus, system (15) has an endemic equilibrium \( \mathcal{E}_N^* = (S_N^*, I_N^*) \) which makes biological sense only when \( \mathcal{R}_N > 1 \). This leads to Theorem 4 below.

**Theorem 4** The alcohol-free endemic equilibrium \( \mathcal{E}_N^* \) exists whenever \( \mathcal{R}_N > 1 \).

The local stability of \( \mathcal{E}_N^* \) is given by the Jacobian evaluated at this point:
\[ J(\mathcal{E}_N^*) = \begin{bmatrix} -\frac{\beta I_N^*}{N_N^*} + \mu & \phi - \frac{\beta S_N^*}{N_N^*} \\ \frac{\beta I_N^*}{N_N^*} & \frac{\beta S_N^*}{N_N^*} - (\mu + \phi) \end{bmatrix}. \quad (18) \]

From (15) we note that \( \frac{\beta S_N^*}{N_N^*} = (\phi + \mu) \), which is obtained from the second equation of system (15) with the right-hand side set equal to zero. Thus, \( J(\mathcal{E}_N^*) \) can be rewritten as
\[ J(\mathcal{E}_N^*) = \begin{bmatrix} -\frac{\beta I_N^*}{N_N^*} + \mu & -\mu \\ \frac{\beta I_N^*}{N_N^*} & 0 \end{bmatrix}. \quad (19) \]

Hence, the characteristic polynomial of the linearised system is given by
\[ a^2 + \left( \frac{\beta I_N^*}{N_N^*} + \mu \right)a + \frac{\beta \mu I_N^*}{N_N^*} = 0, \]
\[ \Rightarrow a_1 = -\mu, \quad a_2 = -\beta(\mathcal{R}_N - 1)\frac{S_N^*}{N_N^*}. \quad (20) \]

Thus, \( \mathcal{E}_N^* \) is locally asymptotically stable for \( \mathcal{R}_N > 1 \). We summarise the result in Theorem 5.

**Theorem 5** The endemic equilibrium \( \mathcal{E}_N^* \) is locally-asymptotically stable whenever \( \mathcal{R}_N > 1 \).

We now study the global stability of system (15) using the Poincaré–Bendixson Theorem [36]. We claim the following result.

**Theorem 6** The endemic equilibrium of the gonorrhea-only model system (15) is GAS in \( \Phi_N \) whenever \( \mathcal{R}_N > 1 \).

**Proof** From model system (15), we note that \( N_N^* = \frac{\Lambda}{\mu} \). Using the fact that \( S_N = N_N - I_N \), it follows that \( S_N^* = \frac{\Lambda}{\mu} - I_N \), and substituting this result into (15), we obtain
\[ I_N^* = \beta I_N^* \left( \frac{\Lambda}{\mu} - I_N \right) - (\phi + \mu)I_N. \quad (21) \]

Using Dulac’s multiplier \( 1/I_N \), it follows that
\[ \frac{\partial}{\partial I_N} \left[ \frac{\β}{\Lambda/\mu} \left( \frac{\Lambda}{\mu} - I_N \right) - (\phi + \mu) \right] = -\frac{\β \mu}{A} = -\frac{\beta}{N} < 0. \quad (22) \]

Thus, by Dulac’s criterion, there are no periodic orbits in \( \Phi_N \). Since \( \Phi_N \) is positively invariant, and the endemic equilibrium exists whenever \( \mathcal{R}_N > 1 \), then it follows from the Poincaré–Bendixson Theorem [36] that all solutions of the limiting system originating in \( \Phi \) remain in \( \Phi_N \), \( \forall t \geq 0 \). Further, the absence of periodic orbits in \( \Phi_N \) implies that the EE of the gonorrhea-only model is GAS whenever \( \mathcal{R}_N > 1 \). \( \square \)

### 3.2.2 Heavy alcohol drinking only endemic equilibrium

This occurs when the entire community consists of heavy alcohol drinkers only, i.e., \( S_N = I_N = 0 \). Using similar analysis as in Sect. 3.2.1, it can easily be shown that the endemic equilibrium is
\[ \mathcal{E}_D^* = \left\{ S_D^* = \frac{\Lambda}{\mu \mathcal{R}_D}, I_D^* = (\mathcal{R}_D - 1)S_D^* \right\} \quad (23) \]

with \( \mathcal{R}_D \) as defined in (6); it follows that the endemic equilibrium \( \mathcal{E}_D^* \) makes biological sense whenever \( \mathcal{R}_D > 1 \). Furthermore, using the analysis done in Sect. 3.2.1, the stability of \( \mathcal{E}_D^* \) can be established. We now discuss the co-existence of heavy alcohol drinkers and non-heavy alcohol drinkers in the community.
Table 1 Model parameters and their interpretations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recruitment rate</td>
<td>( \Lambda )</td>
<td>100 000 yr(^{-1} )</td>
<td>[32]</td>
</tr>
<tr>
<td>Proportion of recruited individuals</td>
<td>( \pi_0, \pi_1 )</td>
<td>0.75, 0.25</td>
<td>Estimate</td>
</tr>
<tr>
<td>Natural mortality rate</td>
<td>( \mu )</td>
<td>0.02 yr(^{-1} )</td>
<td>[32]</td>
</tr>
<tr>
<td>Rate of recovery from gonorrhea after treatment</td>
<td>( \phi )</td>
<td>0.2 yr(^{-1} )</td>
<td>[32]</td>
</tr>
<tr>
<td>Transmission probability for gonorrhea infection</td>
<td>( \beta )</td>
<td>0.22(0.11–0.95) yr(^{-1} )</td>
<td>[23, 32]</td>
</tr>
<tr>
<td>Modification parameter</td>
<td>( \sigma, \eta )</td>
<td>1.2, 1.5</td>
<td>Estimate</td>
</tr>
<tr>
<td>Rate of change in drinking habit</td>
<td>( \alpha, \gamma )</td>
<td>variable</td>
<td>Estimate</td>
</tr>
</tbody>
</table>

3.2.3 Interior endemic equilibrium

This equilibrium refers to the case where both non-heavy alcohol drinkers and heavy alcohol drinkers co-exist. This state is denoted by \( E_{ND}^* \) with

\[
E_{ND}^* = \begin{cases} 
S_N^* &= \frac{\pi_0 \Lambda + \phi I_N^* + \gamma S_D^*}{\lambda_{ND}^* + \mu + \alpha}, \\
S_D^* &= \frac{\pi_1 \Lambda + \phi I_D^* + \alpha S_D^*}{\lambda_{ND}^* + \mu + \gamma}, \\
I_N^* &= \frac{\lambda_{ND}^* S_N^* + \gamma I_D^*}{\mu + \phi + \gamma}, \\
I_D^* &= \frac{\sigma \lambda_{ND}^* S_D^* + \alpha I_N^*}{\mu + \phi + \gamma}, \\
\lambda_{ND}^* &= \frac{(I_N^* + \eta I_D^*)}{N_{ND}^*}. 
\end{cases} \tag{24}
\]

We now study the global stability of the interior equilibrium of system (1) using the Poincaré–Bendixson Theorem (see Appendix).

**Theorem 7** If \( R_{ND} > 1 \), then interior equilibrium \( E_{ND}^* \) is globally asymptotically stable in the interior of \( \Phi \).

Denote the right-hand side of (1) by \( f \) and \( g \) for \( I_N \) and \( I_D \), respectively, and choose a Dulac function as

\[
D(I_N, I_D) = 1/I_D I_N. \tag{25}
\]

Then we have

\[
\frac{\partial (Df)}{\partial I_N} + \frac{\partial (Dg)}{\partial I_D} = -\frac{\gamma}{I_N^2} - \frac{\alpha}{I_D^2} - \frac{\beta \eta}{\Lambda/\mu} \left(\frac{S_N}{I_N^2} + \frac{\sigma S_D}{I_D^2}\right) < 0.
\]

Thus, system (1) does not have a limit cycle in the interior of \( \Phi \). Furthermore, the absence of periodic orbits in \( \Phi \) implies that \( E_{ND}^* \) is GAS whenever \( R_{ND} > 1 \).

3.3 Numerical results

In order to illustrate the results of the foregoing analysis, numerical simulations of model system (1) were carried out using the Matlab programming language and a set of parameter values given in Table 1. Unfortunately, the scarcity of data on gonorrhea and alcohol correlation limits our ability to calibrate; nevertheless, we assume some of the parameters in the realistic range for illustrative purpose. These parsimonious assumptions reflect the lack of information currently available on gonorrhea–alcohol correlation. Reliable data on the risk of transmission of gonorrhea would enhance our understanding and aid in the evaluation of prophylactic measures [22].

**Sensitivity indices of** \( R_{ND} \)

We now investigate the relative importance of different factors responsible for initial disease transmission, which is directly related to the magnitude of \( R_{ND} \). We follow the approach in Chitnis et al. [12]:

(a) \( \frac{\beta}{R_{ND}} \frac{\partial R_{ND}}{\partial \beta} = 1 \),

(b) \( \frac{\sigma}{R_{ND}} \frac{\partial R_{ND}}{\partial \sigma} = \frac{\sigma (\alpha + \mu \pi_1) (\eta k_1 + \gamma)}{(\alpha \eta + k_2) (\gamma + \mu \pi_0) + \sigma (\alpha + \mu \pi_1) (\eta k_1 + \gamma)} \),

(c) \( \frac{\eta}{R_{ND}} \frac{\partial R_{ND}}{\partial \eta} = \frac{\eta [\alpha (\gamma + \mu \pi_0) + \sigma k_1 (\alpha + \mu \pi_1)]}{(\alpha \eta + k_2) (\gamma + \mu \pi_0) + \sigma (\alpha + \mu \pi_1) (\eta k_1 + \gamma)} \).

Results form (26) suggest that parameters \( \beta, \sigma, \eta \) have a positive influence on the magnitude of the reproductive number \( R_{ND} \), that is, an increase (or decrease)
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Fig. 2 Simulations of model system (1) showing the influence of $\alpha$ and $\gamma$ on $R_{ND}$ with parameter values from Table 1. Series B denotes the trend of $R_{ND}$ when $\gamma$ is constant ($\gamma = 0.1$) and $\alpha$ is varying from 0.0 to 0.8 in steps of 0.1, while series C depicts the resulting trend of $R_{ND}$ when $\alpha$ is constant ($\alpha = 0.1$) and $\gamma$ is varying from 0.0 to 0.8 in steps of 0.1. Point A denotes a balance of rate for change in drinking habit

in the magnitude of these parameters will result in an increase (or decrease) in the magnitude of $R_{ND}$. Furthermore, these results suggest that $R_{ND}$ is most sensitive to changes in $\beta$, since an increase in $\beta$ will bring about an increase of the same proportion in $R_{ND}$ and a decrease in $\beta$ will result in a decrease in $R_{ND}$ with a roughly equivalent magnitude. We conclude from this sensitivity analysis that more attention should be paid for reducing unprotected sexual contact ($\beta$) in order to control gonorrhea prevalence. We now use numerical simulations to examine the importance of some of the parameters, namely $\alpha$, $\gamma$, $\phi$, $\pi_0$ (whose analytical results are not included in (26) as they have so many conditions on ascertaining whether they have positive or negative influence on $R_{ND}$) on the transmission and control of gonorrhea epidemic.

We begin by investigating how $R_{ND}$ depends on the change in drinking habit, that is, either one may change from being a heavy alcohol drinker through counselling, poverty, health reasons, tax hike on alcohol beverages, or one may become a heavy drinker due to peer pressure or by choice. We illustrate this dependence in Fig. 2.

Figure 2 is reflecting a large state number of gonorrhea cases for high $\alpha$ and low $\gamma$, suggesting that a change in drinking habit for an individual which results in becoming an alcohol addict may increase the spread of gonorrhea. This is in total agreement with Penkower et al. [35]. While for low $\alpha$ and high $\gamma$ a change in drinking habit resulting in one becoming a non-heavy alcohol drinker may reduce the spread of gonorrhea in the community. The result is in agreement with various studies which suggested that heavy alcohol drinking is related to high risk sexual behaviours which, in return, might result in wide spread of gonorrhea epidemic. Point A denotes a balance of rate for change in drinking habit, and if this occurs in the community, it suggests that besides heavy alcohol drinking other factors are influencing the spread of gonorrhea in the community. We now demonstrate the effect of increase on the percentage of heavy alcohol drinkers in the population.

The following numerical results demonstrate the role of $\pi_1$ and $\phi$ on the dynamics of gonorrhea epidemic.

Numerical results in Fig. 3 suggest that whenever more individuals in the community change their drinking habit to become heavy alcohol drinkers, this may influence the spread of gonorrhea epidemic in the community, but if the reverse is true, with the reproductive number greater than unity, then a number of factors may play a crucial role on the spread of gonorrhea epidemic.

Simulations in Fig. 4(a) suggest that an increase in recruitment of heavy alcohol drinkers in the community will increase the prevalence of gonorrhea cases,
while Fig. 4(b) suggests that an increase in the rate of gonorrhea treatment will have a positive impact on controlling gonorrhea epidemic in the community (especially for high values, as demonstrated in Fig. 4(b)). Overall, the numerical results show that the parameters $\pi_1$ and $\phi$ have contrasting impacts on the magnitude of $R_{ND}$, that is, $\pi_1$ increases the magnitude of $R_{ND}$, while $\phi$ decreases the magnitude of $R_{ND}$, leading to an increase and a decrease of gonorrhea prevalence, respectively. Further analysis of the results suggests that encouraging gonorrhea infectives to seek treatment will be more vital in controlling gonorrhea epidemic in the community.

4 Summary and concluding remarks

A deterministic compartmental model for investigating the effects of heavy alcohol drinking on the transmission dynamics of gonorrhea in the community is formulated and robustly analysed. The epidemic threshold parameter which determines the outcome of the disease is computed and used to assess the dynamics of the disease in the community. Qualitative mathematical analysis of the model has shown that the disease-free equilibrium is globally asymptotically stable whenever the disease threshold parameter is less than unity. Alcohol use increases disease transmission, and prevalence of disease increases with increased rate of becoming an alcohol user. In the event when we have more individuals becoming heavy alcohol drinkers, there is urgent need for intervention strategies such as counselling and educational campaigns in order to curtail the gonorrhea spread.

The proposed study can be extended in various ways by considering some mixing patterns where the
two groups do not mix equally, splitting the population into male/female since there may be some important interactions from this heterogeneity and alcohol dynamics. One can also assume that since the recruitment terms ($\pi_0$ and $\pi_1$) are coming from a similar population, perhaps they should be in balance with the forces swapping between alcohol and non-alcohol, so that $\pi_0$ and $1-\pi_0$ are the equilibrium proportions of the alcohol dynamics $\gamma, \alpha$ and investigate the impact of this on model predictions. This can be done by ignoring the transient dynamics and plotting the equilibrium $I_1, I_2$ and $I_1 + I_2$ against $\gamma, \alpha$.

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**Appendix: Poincaré–Bendixson’s negative criterion**

Consider the nonlinear autonomous system

$$x' = f(x),$$

(27)

where $f \in C^1(\mathbb{R}^n \to \mathbb{R}^n)$. If $x_0 \in \mathbb{R}^n$, let $x = x(t, x_0)$ be the solution of (27) which satisfies $x(0, x_0) = x_0$.

Bendixson’s negative criterion: When $n = 2$, a sufficient condition for the nonexistence of non-constant periodic solutions of (27) is that, for each $x \in \mathbb{R}^2$, $\text{div} f(x) \neq 0$.

**Theorem 8** Suppose that one of the inequalities

$$\mu \left( \frac{\partial f^2}{\partial x} \right) < 0, \quad \mu \left( \frac{-\partial f^2}{\partial x} \right) < 0,$$

holds for all $x \in \mathbb{R}^n$. Then the system (27) has no non-constant periodic solutions.

For a detailed discussion of the Bendixson’s negative criterion, we refer the reader to [17, 36].

**References**

10. Centers for Disease Control and Prevention, Department of Health and Human services, Atlanta: Available at http://www.cdc.gov/alcohol/faqs.htm, last accessed March 11, 2009
38. Tax hike on beer could reduce gonorrhea. Houston Chronicle. Section A. Page 4 Friday 28 April 2000