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Is Relativity’s Equal Footing Treatment of Space and Time Correct in its Entirety?

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Abstract. Both the Special and General Theories of Relativity (hereafter: STR and GTR respectively) put forward by Professor Albert Einstein enjoy an impressive, spectacular, unparalleled and unprecedented success in explaining physical phenomenon as it is revealed by experience. So successful are these theories – so much that – their very foundations are no-longer questioned from a pessimist’s standpoint but from that of an optimist’ viewpoint. Any envisioned or proposed (new) experiment or observational measurement is naturally expected to confirm the STR and GTR. Amongst the foundation stones of the STR and GTR is the treatment of space and time on an equal footing. While in-principle this treatment has no problem when dealing with reference frames, we urge herein that this treat may not be correct when it comes to handling coordinate systems.

Keywords: coordinate system, reference system, relativity: special & general.

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

– Hermann Minkowski (1864 – 1909)

1 Introduction

Both the Special and General Theories of Relativity (hereafter: STR and GTR respectively) put forward by Professor Albert Einstein (1905, 1916) enjoy an ever impressive, spectacular, unparalleled and unprecedented success in explaining physical phenomenon as it is revealed to us by experience (cf. Will 2006, 2009, 2014). So successful are these theories – so much that – their very foundations are no-longer questioned from a pessimist’s standpoint but from that of an optimist’ viewpoint. Any envisioned or proposed new experiment or observational measurement is naturally expected to yield a confirmation of the STR and GTR. Should a measurement contradict these theories, this would come as nothing short of a Revolution in Science. This is the confidence with which the theories of relativity are trusted by scientists today.

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Be that it may – herein, we do not rail against the theory of relativity, but merely ask if one of relativity’s central assumptions on its treatment of the time dimension is entirely correct. As is well known, the theory of relativity treats time on an equal footing with space and this treatment extends to transformations between different coordinate and reference systems. Our borne of contention is on this treatment being extended to transformations between different coordinate systems – on this, we ask, “Is this extension correct?” We point out that this equal footing treat of time may not be correct when it comes to coordinate transformations. In order that we put our point across in-as-clear a manner as is possible, in §2 we define the problem and in §3.1 and 3.2, we find ourself having to do the most trivial thing – that is – we shall define what it is we call a Coordinate System and a Reference System.

We already know from the STR that time does not transform absolutely when dealing with different reference systems and this was Professor Einstein’s radical new idea that changed forever our view of time and the order of the natural World. Prior to Professor Einstein’s radical revision, Sir Isaac Newton had defined time an invariant and absolute physical quantity existing independent of anything external; of it, he declared in his famous book Philosophae Naturalis Principia Mathematica of 1687:

“Absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration: relative, apparent and common time, is some sensible and external (whether accurate or unequal) measure of duration by the means of motion, which is commonly used instead of true time.”

That is to say, according to Sir Isaac Newton, absolute time exists independently of any perceiver in the Universe and it passes or progresses at a constant and consistent pace throughout the Universe. Unlike relative time, Sir Isaac Newton believed absolute time was imperceptible and could only be understood from mathematical stand point. Accordingly, we humans are only capable of perceiving relative time, which is a measurement of perceivable objects in motion like the when a car moves from one point to the other, it is from these movements, we infer the passage of time.

We should say, we are not about to change Professor Einstein’s new view of time but simple “clip the wings” of our use of this idea when dealing with coordinate systems (not reference systems). We ask here the question whether or not the time coordinate is invariant under a change of the coordinate system? The answer to this question shall provide an answer to the question of whether or not it is right that a change of the coordinate system should lead to a change in the physics as happens in black-hole physics where a singularity can be smothered out by a change of the coordinate system. In conclusion, we shall establish that time – viz, when transforming between different coordinate systems, it (time) is a scalar quantity and this manifests itself as a self-evident-truth beyond any doubt whatsoever.

2 Problem

Let us look closely at the transformation law:

\[ \Delta x'^\mu = \frac{\partial x'^\mu}{\partial x^\mu} \Delta x^\mu. \]  \hspace{1cm} (1)
This transformation law is equally applied when dealing with coordinate and/or reference systems. From it, we pluck-out the time coordinate, i.e., \( \mu' = \mu = 0 \), i.e.:

\[
\Delta t' = \frac{\partial t'}{\partial t} \Delta t + \frac{\partial t'}{\partial x} \Delta x + \frac{\partial t'}{\partial y} \Delta y + \frac{\partial t'}{\partial z} \Delta z.
\]

From pure logic, one knows very well that changing the way we label points in space should not alter the geometry of space nor the flow of time between the previous labels and the new labels – this is not debatable but an invariable fact of logic, intuition and physical reality. For example, changing the name of a street should not physically alter the street itself. However, from (2), it follows that a time difference of \( \Delta t' \) in the primed coordinate system is related to the time lapse \( \Delta t \) in the un-primed coordinate system such that \( \Delta t' \neq \Delta t \). Changing the coordinate system should never ever alter time, the meaning of which is that we must \( \Delta t' = \Delta t \). This is only possible if-and-only-if:

\[
\frac{\partial t'}{\partial t} \equiv 1, \quad \frac{\partial t'}{\partial x} = \frac{\partial t'}{\partial y} = \frac{\partial t'}{\partial z} \equiv 0.
\]

With (3) as given, time will flow equably between the two coordinate systems, but if (3) does not hold identically, then this means for different coordinate systems, time moves at different rates! That is to say, if say one is somewhat fade up of say the way time behaves in their rectangular spacetime coordinates \((x, y, z, t)\), they can choose say a spherical system of coordinates \((r, \theta, \phi, \tilde{t})\) where time \((t, \tilde{t})\) between these two coordinate systems flows un-equably and in a manner that best suits their desideratum?!

A very good example of this is in blackhole physics where the Schwarzschild singularity is treated not as a physical singularity, but as a mathematical singularity arising only as a consequence of the choice of coordinates employed. For example, to try and solve this issue of the Schwarzschild singularity, Eddington (1924) and Finkelstein (1958), proposed that the Schwarzschild singularity can be whisked (transformed) away by moving from one set of spherical coordinates \((r, \theta, \phi, t)\) to a new set of spherical coordinates \((r, \theta, \phi, \tilde{t})\), if we transformed the time coordinate from \(t\) to a new set of time coordinate \(\tilde{t}\).

### 3 Coordinate and Reference Systems

It is necessary that we define succinctly what we mean by reference system and coordinate system – these two are used interchangeably in most textbooks of physics. While this is a trivial thing, the understanding of what is a reference system and coordinate system is key to the present presentation. For example, starting with the Schwarzschild metric; Stephani (2004, p.304), in his effort of trying to describe events near and at the event horizon of in blackhole, goes on to say “We seek coordinate systems which are better adapted to the description of physical processes ...”. This is nothing more than an admission that physics in different coordinate systems will be different – there exists coordinate systems that are unsuitable for the description of physical events. Why should this be so? Physics and or physical processes should never be dependent on the choice of coordinates – at the very least, this is in conflict with the seemingly sacrosanct Principle of Relativity. Let us devote some little time to understanding what is a coordinate system and a reference system and thereafter look deeper into the meaning of what these really are. **Prima facie**, this exercise to make an introspection of what a coordinate system and a reference system really are may appear naïve,
nonetheless, we believe it is a necessary and very important exercise for the development and evolution of thought regards the fundamentals of relativity.

3.1 Reference System

Shown in Figure (1) are two reference systems, the primed and the unprimed each with their respective $x$, $y$ and $z$ axis. For any given coordinate system (e.g. rectangular, spherical, curvilinear etc) there exists a point that one can call the point of origin, this point can be any-point, there ought not to be a preferred point. In the usual three dimensions of space, this point is the point $(0,0,0)$ – this choice gives the easiest way to manipulate the coordinates. Once the observer has set the $(0,0,0)$ point, they will set up about this point $(0,0,0)$, their axis and the set of axis then constitutes the Reference System. The observer that has declared their point of origin and has set their reference system “sees” every other point relative to the $(0,0,0)$ point thus this point is their point of reference which together with the set of axis is in the usual language of STR is the reference system. The reference system thus provides one with a reference point $(0,0,0)$ and a set of axes relative to which that particular observer can measure the position and motion of all other points in spacetime as seen in other reference systems. If another observer has their reference system and this system is observed by another observer as moving, then, these two observers, as shown on Figure (1); are in relative motion.

In simple terms, the above – in our view; defines a reference system and we hope the reader will be able to make a clear distinction between a coordinate system and reference system once we have defined a coordinate system in §3.2.

The STR is concerns itself with the nature of Physical Laws under a change of the reference system, i.e., from one-point of spacetime to another depending on these points’s state of motion relative to one another, while the GTR is concerned with nature of Physical Laws under both a change of the coordinate system and reference system. The STR posits that the Laws of Physics remain the same for observers in uniform relative motion with the GTR positing through the Principle of Equivalence that even for observers in uniform relative acceleration the Laws of Physics remain the same and these are the same as for those observers.
in uniform relative motion. The GTR goes further and extends this to encamps different coordinate systems by maintaining that the Laws of Physics remain invariant under a change of coordinate system. We will point out in §3.2, a logical flaw in the GTR in its endeavours to be a beacon and paradigm that describes Natural Laws under general coordinate and reference system transformations. This is deeply rooted in its treatment of time under a change of the coordinate system (NB: not reference system). The logical flaw lies in the equal-footing treatment of the space and time coordinates applicable to the STR or to transformations between different but equivalent reference systems being unconsciously extended to describe natural processes under a change of the coordinate system.

### 3.2 Coordinate System

When thinking about space, it is extremely useful to think of it as constituting of points, each labelled so that one can distinguish one point from another, each point is and must be unique. These labels are called coordinates. One must choose these labels in such a way that it is easy to manipulate them. In practice, numbers are used because we understand and can manipulate them – actually, we do not think there is any other way of labelling space besides using numbers. To manipulate these labels (numbers), a universal and well defined rule must be set out so as to label and manipulate the labels and this is what is called the Coordinate System. One ought to be free to choose any coordinate systems of their choice provided the labelling scheme makes each point to be unique because any space exists independent of the coordinate systems used.

Examples of coordinate systems are the spherical coordinates (denoted: \( r, \theta, \varphi \)), rectangular (denoted: \( x, y, z \)), cylindrical (denoted: \( r, \theta, z \)) and curvilinear (denoted: \( h_1, h_2, h_3 \)) to mention but a few. The coordinate itself is thought to have no physical significance but only its relative distance from other coordinates is what is of physical significance. Due to Professor Herman Minkowski (1864 – 1909)’s brilliant insight, we must add a forth dimension (\( t \)) in order to label the arena where physical events take place \( i.e., \), for spacetime where spherical coordinates are used to label space, we have \( (r, \theta, \varphi, t) \), and likewise for rectangular spacetime.
coordinates we have \((x, y, z, t)\) etc. The question is, for example when we have to make a transition from say rectangular spacetime coordinates to say spherical spacetime coordinates \((r, \theta, \varphi, t)\), do we have the right to alter the forth dimension? We shall provide an answer to this in a short-while.

The rectangular \((x, y, z)\) and spherical \((r, \theta, \varphi)\) coordinates together with their differentials \((dx, dy, dz; dr, d\theta, d\varphi)\) are related by the following equations:

\[
\begin{align*}
    x &= r \cos \theta \sin \varphi \\
y &= r \sin \theta \sin \varphi \\
z &= r \cos \varphi
\end{align*}
\]

\[
\begin{align*}
    dx &= \cos \theta \sin \varphi dr - r \sin \theta \sin \varphi d\theta + r \cos \theta \cos \varphi d\varphi \\
y &= \sin \theta \sin \varphi dr + r \cos \theta \sin \varphi d\theta + r \sin \theta \cos \varphi d\varphi
\end{align*}
\]

\[ (4) \]

In matrix notation, the differentials are given by:

\[
\begin{pmatrix}
    \Delta x \\
    \Delta y \\
    \Delta z
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta \sin \varphi & -r \sin \theta \sin \varphi & r \cos \theta \cos \varphi \\
    \sin \theta \sin \varphi & r \cos \theta \sin \varphi & r \sin \theta \cos \varphi \\
    \cos \varphi & 0 & -r \sin \varphi
\end{pmatrix}
\begin{pmatrix}
    \Delta r \\
    \Delta \theta \\
    \Delta \varphi
\end{pmatrix}. \tag{5}
\]

The above \([4]\) is a transformation from rectangular to spherical space-coordinates. If we decided to add the fourth dimension and thereafter proceed to give it an equal status with the 3-space dimensions, then \([4]\) will become:

\[
\begin{pmatrix}
    \Delta x \\
    \Delta y \\
    \Delta z \\
    \Delta t'
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta \sin \varphi & -r \sin \theta \sin \varphi & r \cos \theta \cos \varphi & \Lambda_{03} \\
    \sin \theta \sin \varphi & r \cos \theta \sin \varphi & r \sin \theta \cos \varphi & \Lambda_{13} \\
    \cos \varphi & 0 & -r \sin \varphi & \Lambda_{23} \\
    \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33}
\end{pmatrix}
\begin{pmatrix}
    \Delta r \\
    \Delta \theta \\
    \Delta \varphi \\
    \Delta t
\end{pmatrix}, \tag{6}
\]

where \(\Delta t'\) is the differential time associated with the rectangular coordinate and \(\Delta t\) is the differential time associated with the spherical coordinates. If we are to treat space and time in these transformations on an equal footing, then, in general \((\Lambda_{03} \neq 0; \Lambda_{3\mu} \neq 0)\).

In our treatment of space and time on an equal footing, one thing that is clear and all will agree is that the space transformation, are not mix with the time dimension. For example, if \((\Lambda_{01} \neq 0)\), then:

\[
\Delta x = \cos \theta \sin \varphi \Delta r - r \sin \theta \sin \varphi \Delta \theta + r \cos \theta \cos \varphi \Delta \varphi + \Lambda_{03} \Delta t. \tag{7}
\]

First and foremost, what the above \([7]\) really means is that, the coordinate \(x\) (hence \(y\) and \(z\)) is time-dependent. Coordinates \((i.e.\) space labels) can not be time dependent. In-order that we conform to this absolutely necessary requirement, clearly, we must have:

\[
\Lambda_{\mu 3} = \Lambda_{3\mu} = 0, \quad \text{for} \quad \mu \neq 3, \tag{8}
\]

that is to say:

\[
\begin{pmatrix}
    \Delta x \\
    \Delta y \\
    \Delta z \\
    \Delta t'
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta \sin \varphi & -r \sin \theta \sin \varphi & r \cos \theta \cos \varphi & 0 \\
    \sin \theta \sin \varphi & r \cos \theta \sin \varphi & r \sin \theta \cos \varphi & 0 \\
    \cos \varphi & 0 & -r \sin \varphi & 0 \\
    0 & 0 & 0 & \Lambda_{33}
\end{pmatrix}
\begin{pmatrix}
    \Delta r \\
    \Delta \theta \\
    \Delta \varphi \\
    \Delta t
\end{pmatrix}. \tag{9}
\]

The above \([9]\) leaves \((\Lambda_{33} \neq 0)\), and this invariably translates to \((\Delta t' = \Lambda_{33} \Delta t)\), that is to say \((\Delta t' \neq \Delta t)\). For all we know, changing coordinates must not (and should never ever)
lead to a change in the time interval. Whether I am using rectangular, spherical or whatever coordinate system, a time interval of 5 min remains 5 min no matter the coordinate system I decide to switch to. Otherwise, having \(\Lambda_{33} \neq 0\), implies that one can alter (just by thinking about it and nothing else) their own time by changing the way they wish to label space \(i.e.\) if \(\Lambda_{33} \neq 0\), we have – without any physical process but just the way we decide to think of space; time dilation intimately associated with the way in which we label points in spacetime?!

Surely, herein lies one of central problems of the GTR for:

\[
\text{This means a photon can be blue or red-shifted by just changing the system of coordinates (and not the reference system)!}
\]

Red or blue-shifting is a physical process (which can arise in the transformation of the reference system) but changing of the system of coordinates is not a physical process at all! Here we have it - this is the source of our problems in our endeavours to completely understand \textit{Nature} from the current GTR view-point especially when it comes to blackholes, we alter the time-coordinate so as to read ourself of singularities under the guise of legitimate coordinate transformations; but in so doing we are making a physical alteration and not an alteration of the way we label spacetime.

Clearly, the only way in which a photon’s physical state will remain invariant is if time preserved its nature under a change of the coordinate systems. This could mean time is not a vector but a scalar when it comes to coordinate transformations. Under reference system transformations, yes time is a vector, it is part and parcel of the four vector. If time behaved as predicted by equation \(\Delta t' = \Lambda_{33} \Delta t\) with \(\Lambda_{33} \neq 0\), it could mean all physical events in spacetime are affected by a change of the coordinate system and as already stated, it means the way in which we label points does has a realizable physical significance?!

This on its own makes no physical or logical sense, it surely constitute a serious anti-desideratum – for it would allow for pure magic to occur in physics \(i.e.\), one would choose at will just by thinking about it, a coordinate systems of their liking and they would give a different description from that of another observer that employs a different set of coordinates of the same physical phenomena or event in spacetime. A priori to this analysis and also a posteriori justified execution, is that, it is absolutely necessary that we put forward the following \textbf{Protection Postulate}:

\textbf{Postulate I:} \textit{In order to preserve the physical state and the chronological evolution of a physical system when making a transition from one coordinate system to another, of itself, and from its own nature, time must flow equable without relation to anything external, it must remain invariant under any kind of transformation of the coordinate system.}

That is to say:

\[
\Delta t' = \Delta t,
\]

hence:

\[
\Lambda_{33} = 1.
\]

It is not difficult to show that if a particular or all spatial coordinates were to transform in a non-linear manner with respect to the corresponding coordinate, events and or points in
spacetime will cease to be unique and also the physics is altered just by changing the coordinate system! In order to strictly preserve the physics and second to preserve the uniqueness of events when a transition to a new coordinate system is made, it is necessary to put forward another protection postulate:

**Postulate II:** In order to preserve the physics when a transition to a new coordinate system is made, and for this same transition to preserve the uniqueness of physical events in spacetime, the points in the new coordinate system for a non-periodic coordinate system, must be linear and have a one-to-one relation with the old one and in the case of a periodic coordinate system the periodicity is ignorable.

In Postulate I, linearity has a two-fold meaning here:

1. Suppose in a transformation of the coordinate system from A to B, a point in the coordinate system A has more than one corresponding coordinate for a non-periodic coordinate system (e.g. a spherical coordinate system is a periodic coordinate system, this periodicity can be ignored because it does not physically place the point to another point in the same space), then, in such a coordinate transformation, events cease to be unique; this must be guarded against, hence the second postulate.

2. Mathematically speaking, the first postulate means that when it comes to coordinate transformations, time is a scalar quantity, i.e., for a coordinate transformation and not a transformation of the reference system:

\[
\frac{\partial x^0}{\partial x'^0} \equiv \frac{\partial x'^0}{\partial x^0} \equiv 1, \quad \text{and} \quad \frac{\partial x^i'}{\partial x^0} \equiv \frac{\partial x^i}{\partial x'^0} \equiv \frac{\partial x^i}{\partial x^0} \equiv \frac{\partial x^i'}{\partial x'^0} \equiv 0.
\]

(12)

In this way, by way of arguing, we thus have established here that time must behave as a scalar when transforming from one system of spacetime coordinates to another and this is not so when transforming from one reference system to another. Because of this, let us adopt the terminology *coordinate scalar* or *coordinate vector* to mean a quantity behaves as a scalar under a coordinate transformation and likewise we will have a *reference scalar* and *reference vector* to mean a quantity that transforms as a scalar or vector when transforming from one reference system to the other.

4 **General Discussion**

The GTR was introduced along with the principle that there exists no system of coordinates that is preferred, but that any arbitrary system of coordinates would do. In-order for one to be able to find an arbitrary system of coordinate, the GTR introduced the complex mathematical machinery of covariance which amongst others requires that the Laws of Physics be expressible in tensor form, that is to say, any of the Laws of Physics – not just in the GTR.

We have argued herein that when it comes to the time dimension, the equal footing of treatment of time and space is erroneous when it comes to transformations involving a change of the coordinate system as this is tantamount to a physical alteration of the state of a physical system. What motivated this reading is that, because of the general equal footing treatment, one finds in some textbooks and research papers where the time dimension is tempered with when making coordinate transformations. As argued herein, this is equivalent to magic – *albeit*, magic that can not and does not materialize in the real world but remains only confined to the theorist’s mind.
If the present proposal is accepted, then, Sir Isaac Newton’s treatment of time as an invariant is the correct way to handle time when making coordinate transformations but when it comes to transitions between reference systems, Professor Einstein’s treatment carries the day. So, both Sir Isaac Newton and Professor Einstein’s treatments are correct in one way and at the same, these treatments not correct.

5 Conclusion

When it comes to transformations of the system of coordinates, the time coordinate is to remain invariant and the resulting transformations of the space coordinates are to be completely independent of time – they must contain no time dependence whatsoever. Transformation of the time coordinate during a coordinate transformation and the introduction of time dependence into the space coordinates is tantamount to a physical alteration of the system under consideration yet a coordinate transform by its very nature, it is not a physical alteration but merely a different way of labelling the points of space.

References


