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Apsidal Precession of the Outer Solar Planetary Orbits due to the Pioneer Anomaly

G. G. Nyambuya

Abstract Despite the now common position that the Pioneer anomaly is not a real gravitational effect but an effect due to the on-board thermal recoil forces – for curiosity’s sake, we here take the suggestion of Nyambuya (2015) where it has been assumed that the Pioneer anomaly – can, in-principle, be attributed to a gravitational effect due to these spacecrafts accreting some material from a rarefied Interplanetary Medium (IPM) in the domain where the Pioneer anomaly has manifested \([20 \text{ AU} \lesssim r \lesssim 70 \text{ AU}]\). If this assumption is correct, then, the expected Pioneer acceleration of these spacecrafts may be much smaller than the Pioneer acceleration to cause as noticeable apsidal precession of the outer Solar planets Uranus, Neptune and Pluto, thus making it difficult to rule out a gravitational origin of the Pioneer anomaly.

Keywords anomaly: Pioneer – precession: apsidal, perihelion

1 Introduction

The now more than 30-year-old problem of the Pioneer anomaly has attracted considerable attention in the last 10 – 15 years or so (see e.g. Nieto and Turyshev 2004). This anomaly is an unmodeled sunward acceleration of magnitude \(a_P\) of the Pioneer spacecrafts which were launched by the United States of America (USA)’s National Aeronautic Space Administration (NASA) on March 3, 1972 and March 6, 1973 – respectively; as probe missions to prepare for the then coming Voyage missions (Nieto and Turyshev 2004, p.13):

\[
 a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ ms}^{-2}. \tag{1}
\]

The anomalous Pioneer signal began to manifest itself when these spacecrafts where approaching the distance of \(\sim 20 \text{ AU}\) from the Sun; the signal is confirmed to be present up to about \(\sim 70 \text{ AU}\) from the Sun’s center.

This anomaly was first noticed around 1980 (Nieto and Turyshev 2004) but only taken seriously in the late 1990s by Anderson et al. (1998). To this day, there is no generally or universally accepted explanation to its cause; we must at this point hasten and say that, Turyshev and Toth (2011a); Turyshev et al. (2012)’s proposal for on-board thermal recoil forces is now taken as the de facto solution to this problem. This does not however close the chapter of the search for alternative causes.

An acceleration of the order of \(10^{-10} \text{ ms}^{-2}\), may appear minute, so much that one may consider it an artefact or noise in the signal. Perhaps, this is the reason it took such a long-time to take this anomalous signal seriously. Be that it may, several intensive studies have since dismissed this view. There is also the possibility of observational errors, which include measurement and computational errors, leading to a situation where these observational errors being interpreted as an anomaly in the motion of the spacecrafts. According to Turyshev (2007), further analysis of the Pioneer anomaly data has shown that significant errors are not likely because several independent analyses have shown the existence of the Pioneer anomaly as a real effect, and not an artefact, noise, observational error nor an error in the Orbit Determination Program (ODP). In line with the view of the afore-stated researchers, we take their same line of reasoning, namely that, the anomaly is present and real; and thus needs an explanation.

Assuming the Pioneer anomaly is caused by a force field (let us call it the Pioneer force: \(F_P = -ma_P\)) like the Newtonian gravitational field – in this reading – we want to know, what effect – if any will, the Pioneer force have on the orbits of planets. In particular, we want to know the contribution this force has on the anomalous apsidal precession of planets. The planets that are to be affected by this force are the planets, Uranus, Neptune and the dwarf planet Pluto. These planets lie in the zone \([20 \text{ AU} \lesssim r \lesssim 70 \text{ AU}]\) where

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the Pioneer anomaly has manifested itself. The inner planets in the region \( r < 20 \) AU are not expected to be affected by the Pioneer force field.

In-closing this introductory section, we give a synopsis of the present letter, it is organised as follows: in \( \S 2 \), for instructive purpose, we give an exposition of Einstein’s calculation of the apsidal precession of the planet Mercury. Thereafter, we present a calculation for the apsidal precession due to the Pioneer force field and lastly, in \( \S 4 \) and (5), we give a general discussion and conclusion drawn thereof.

2 General Relativistic Anomalous Apsidal Precession

When Einstein (1915c) applied his newly discovered GTR (Einstein 1915a,b) to the problem of the precession of the perihelion of the planet Mercury he obtained that the trajectory of a planet orbiting a central body of mass \( M \) must be described by the equation:

\[
\frac{d^2u}{d\varphi^2} + u - \frac{1}{l} = \left(\frac{3GM}{c^2}\right) u^2,
\]

where is the semi-latus rectum of the orbit, \( u = 1/r \) with \( r \) being the radial distance of a test body from the central massive body, \( \varphi \) is the angular displacement in the \( \hat{\varphi} \)-direction, \( G \) is the usual Newtonian constant of gravitation. For instructive purposes, we will give a detailed calculation leading to the final result of Einstein formula for the apsidal precession.

We shall use a perturbation method to arrive at the desired solution. To that end, we shall assume that \( u = \eta_E + u_p \), where \( u_N = (1 + e \cos \varphi)/l \) is the usual Newtonian solution where \( e \) is the eccentricity of the orbit and \( \varphi \) is the angular displacement. The task at hand is to obtain the particular solution \( u_p \). To this, we shall begin by substituting \( al = 1 + e \cos \varphi + u_p/l \) into, (2); so doing, one obtains:

\[
\frac{d^2u_p}{d\varphi^2} + u_p = \frac{\eta_E}{l} \left(1 + e \cos \varphi + u_p/l\right)^2.
\]

where \( \eta_E = 3GM/lc^2 \) is an extremely small quantity – albeit, important in the explanation of the anomalous apsidal precession and the secular rate of a test body about its central massive gravitating body.

To obtain Einstein’s formula for the apsidal precession, the term \( u_p/l \) on the right hand side of (3) is dropped in the assumption that \( 1 + e \cos \varphi \gg u_p/l \) and further, the first order approximation \( \left(1 + e \cos \varphi\right)^2 \sim 1 + 2e \cos \varphi \) is made. Proceeding along these lines one obtains from this set of approximation, the following:

\[
\frac{d^2u_p}{d\varphi^2} + u_p = \frac{\eta_E}{l} \left(1 + 2e \cos \varphi\right).
\]

The solution to (4) is:

\[
u_p = \frac{\eta_E(1 + e \varphi \sin \varphi)}{l}
\]

hence, the full solution \( u = u_N + u_p \) is therefore:

\[
u = \frac{1 + e \cos \varphi}{l} + \frac{\eta_E(1 + e \varphi \sin \varphi)}{l}
\]

\[
= \frac{1 + e(\cos \varphi + \eta_E \varphi \sin \varphi)}{l}
\]

\[
\approx \frac{1 + e(\cos \varphi + \eta_E \varphi \sin \varphi)}{l}
\]

For small \( \eta_E \):

\[
\cos \varphi + \eta_E \varphi \sin \varphi \approx \cos(\eta_E \varphi) \cos \varphi + \sin(\eta_E \varphi) \sin \varphi
\]

\[
= \cos(\gamma_E \varphi + \varphi)
\]

\[
= \cos[(1 + \eta_E)\varphi]
\]

\[
= \cos(\gamma_E \varphi)
\]

where \( \gamma_E = 1 + \eta_E \); therefore:

\[
u = \frac{1 + e \cos(\gamma_E \varphi)}{l}
\]

With this equation, we are now ready to derive Einstein’s formula for the apsidal precession.

At the perihelion we will have: \( \gamma_E \varphi = 2n\pi \) and this implies: \( \varphi = 2n\pi \gamma_E \sim 2n\pi - 6n\pi G M/l c^2 \). Essentially, this means that the perihelion advances by \( \Delta \varphi = 6nG M/l c^2 \) per revolution, hence – the average the rate of precession of the perihelion is given by:

\[
\frac{\Delta \varphi}{T_\varphi} = \frac{6 \pi G M}{T_\varphi c^2 (1 - e^2)} \approx \xi_{GTR},
\]

where \( T_\varphi \) is the time period of revolution for the orbital motion under consideration. This is Einstein’s formula derived in 1916 soon after he discovered the GTR. After deriving this beautiful result which gave the GTR impetus and a good head-star because of its “seemingly tailor made” prediction of the perihelion precession result for Mercury, Einstein concluded:

“Calculation gives for the planet Mercury a rotation of the orbit of 43” per century, corresponding exactly to the astronomical observation (Leverrier); for the astronomers have discovered in the motion of the perihelion of this planet, after allowing for disturbances by the other planets, an inexplicable remainder of this magnitude.”
For planet Mercury \(T_w(\varphi) = 0.24 \text{ yrs}\), \([e(\varphi) = 0.206]\) and \([R_{\text{min}}(\varphi) = 0.20 \text{ AU}]\), the reader can verify that inserting these values into (9) leads to \(\sim 43''\) per century.

### 3 Pioneer Anomaly

Viewed from a Newtonian gravitational standpoint, Einstein’s equation (2) adds a correction term \(F_E(u)\) to the Newtonian paradigm: \(F_E(u) = -3me^2/(GMc^2)^2u^4\). If we in-cooperate the Pioneer force field \(F_P(u) = -ma_P\) as an additional force, then, the resultant equation for the orbit is:

\[
\frac{d^2u}{d\varphi^2} + u = -F_N(u) + F_E(u) + F_P(u).
\]

We can write this equation as:

\[
\frac{d^2u}{d\varphi^2} + u - \frac{1}{l} = \left(\frac{3GM}{c^2}\right)u^2 + \left(\frac{a_P}{J_{\varphi}^2}\right)u^{-2}.
\]

As in the Einstein case in the previous section, we assume a solution of the form \(u = u_N + u_p\), where the task at hand is to determine the particular solution \(u_p\). Substituting \(u = u_N + u_p\) into (11), we obtain:

\[
\frac{d^2u_p}{d\varphi^2} + u_p = \left(\frac{3GM}{c^2}\right)(1 + e \cos \varphi + u_p l)^2 + \left(\frac{l^2 a_P}{J_{\varphi}^2}\right)(1 + e \cos \varphi + u_p l)^{-2}.
\]

Again, as in the Einstein case in the previous section, we shall assume that \((e \cos \varphi + u_p l) \ll 1\), the meaning of which is that to first order approximation, the following approximations will hold \([(1 + e \cos \varphi + u_p l)^2 \sim 1 + 2e \cos \varphi]\) and \([(1 + e \cos \varphi + u_p l)^{-2} \sim 1 - 2e \cos \varphi]\). These approximations will reduce equation (12) to:

\[
\frac{d^2u_p}{d\varphi^2} + u_p = \frac{\eta_E(1 + 2e \cos \varphi)}{l} + \frac{\eta_P(1 - 2e \cos \varphi)}{l},
\]

where \((\eta_p = l^2a_P/J_{\varphi}^2)\). The exact solution to (13) is:

\[
u_p = \frac{\eta_E(1 + e \varphi \sin \varphi)}{l} + \frac{\eta_P(1 - e \varphi \sin \varphi)}{l},
\]

hence the full solution is given by:

\[
u = \frac{1 + \eta_E + \eta_P + e \cos \varphi + e\eta_E \varphi \sin \varphi - e\eta_P \varphi \sin \varphi}{l}.
\]

From the approximation (7), equation (15) will reduce to:

\[
u = \frac{1 + e \cos(\gamma_{EP} \varphi)}{l},
\]

where \((\gamma_{EP} = 1 - \eta_E + \eta_P)\). From this, it is clear that the resulting apsidal precession rate will be given by:

\[
\left\langle \frac{\Delta \varphi}{T_\varphi} \right\rangle_E = \left\langle \frac{\Delta \varphi}{T_\varphi} \right\rangle_E + \left\langle \frac{\Delta \varphi}{T_\varphi} \right\rangle_P,
\]

where:

\[
\left\langle \frac{\Delta \varphi}{T_\varphi} \right\rangle_P = \frac{-2(1 - e^2)^2a^2a_P}{T_\varphi GM} = \dot{\omega}_P,
\]

is the Pioneer force field contribution to the anomalous apsidal precession rate of planetary orbits. The self-explanatory Table (1) lists the expected anomalous apsidal precessions due to the Pioneer force field.

**Table 1** Predicted Anomalous Apsidal Precessions due to the Pioneer Force Field

<table>
<thead>
<tr>
<th>Object</th>
<th>(a)</th>
<th>(T_w)</th>
<th>(e)</th>
<th>(\dot{\omega}_{\text{GTR}})</th>
<th>(\dot{\omega}_P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranus</td>
<td>19.2</td>
<td>84.0</td>
<td>0.0473</td>
<td>+0.0024</td>
<td>-80.00 \pm 10.00</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.0</td>
<td>164.8</td>
<td>0.0086</td>
<td>+0.0008</td>
<td>-100.00 \pm 20.00</td>
</tr>
<tr>
<td>Pluto</td>
<td>39.5</td>
<td>248.5</td>
<td>0.2488</td>
<td>+0.0004</td>
<td>-100.00 \pm 20.00</td>
</tr>
</tbody>
</table>

### 4 General Discussion

The predicted apsidal precession due to the Pioneer force field are not only large, but in the opposition direction to what is predicted by the GTR. Using EPM2008 ephemerides data set, according to Iorio (2015) in a private communication with astronomer E. V. Pitjeva of the Russian Institute of Applied Astronomy; Pitjeva (2010) gives for Uranus \((-3.89 \approx 3.90''\text{cy}^{-1})\) and for Neptune and Pluto, she gives \((-4.44 \pm 5.40''\text{cy}^{-1})\) and \((-2.84 \pm 4.91''\text{cy}^{-1})\) respectively. This obvious rules out the Pioneer anomaly as a gravitational effect.

Alas! What Pitjeva (2010)’s preliminary results (which are compatible to zero given that the errors are larger than
the actual measured result) suggest is that – the Pioneer anomaly, if it is a real physical field effect due to a force field emanating from – say – the Sun, it must have a different values for these outer planets. Additionally, despite the compatibility with a null result – the negative sign associated with these anomalous apsidal precessions of Pitjeva (2010), is that they can not be general relativistic in nature since the GTR is clear on the sign – it must be positive! This now takes us to our own suggestion – Nyambuya (2015) – as to the cause of the Pioneer anomaly.

There-in the reading Nyambuya (2015), it has been suggested that – perhaps – the Pioneer anomaly is due to the gravitational effect of some smeared-out Interstellar Medium (IPM) where-in, these spacecrafts accrete from this IPM as at a rate of \( 6.00 \times 10^{-4} \text{ kg/yr} \) as they plough through it as their radial speeds of \( \sim 12 \text{ km/s} \). The is to say, if \( m \) is the mass of the object ploughing through this IPM at a radial speed \( v \), and \( \dot{m} \) is its accretion rate of this object, then according to Nyambuya (2015), \( a_P \) is such that:

\[
a_P = - \left( \frac{\dot{m}}{m} \right) v. \tag{19}
\]

Therefore – it follows from this – that, if indeed the Pioneer anomaly is due to mass accretion of the Pioneer spacecrafts in the rarefied IPM, then – for any statistically significant negative extra-anomalous apsidal precession of the outer planets, or, of any orbiting object in the Pioneer region \([20 \text{ AU} \lesssim r \lesssim 70 \text{ AU}]\), not only can a gravitational origin of this effect be attributed to it, but one can compute from (18) the requisite value of \( a_P \) and proceed to use this value in-conjunction with equation (19) to estimate \( v \) or \( \dot{m}/m \). The value of \( v \) could give the secular drift of these planets. This secular drift is similar to the secular drift of mean Earth-Moon distance that has been measured by independent group of American and Russian astronomers Standish (2005); Krasinsky and Brumberg (2004); Pitjeva and Pitjev (2012) where they find a value of \( \sim 7 - 15 \text{ cm/yr} \).

Accurate observations of the anomalous apsidal precession observations of the outer planets – Uranus, Neptune and Pluto, are currently not available and this is due to their long orbital periods. However, according to Iorio (2012), there is some ray of hope regarding Planet Uranus which is the only outer planet having completed a full orbital revolution over the time span for which modern observations are available. Apart from outer planets, one can also study (as suggested e.g. by Page et al. 2006) the motions of asteroids laying beyond 20 AU such as asteroid 5335 and 1995 SN55 laying at radial distances of 20.8 AU and 38.4 AU respectively.

While the present and dominate view is that the Pioneer anomaly is due to a thermal effects (Turyshcev and Toth 2011a; Turyshcev et al. 2012), it is important that alternative explanations are found until such a time that a new, perhaps dedicated space probe detects this anomalous acceleration. As suggested herein, planetary ephemerides may offer lasting solutions to this issue should statistically significant negative extra-anomalous apsidal precessions of the outer planets become available. We believe that this method (of checking the anomalous apsidal precession rate) needs to be applied to planets or stellar bodies whose entire orbit (not only the perihelion) lays beyond 20 AU. These inner planets – Mercury, Venus, Earth and Mars, show a negative unaccounted apsidal precession (see e.g., Iorio 2009), and as just said, the Pioneer force field can not be the cause of these as it does not operate in this domain.

5 Conclusion

Assuming the correctness of what has been presented here, we hereby make put forward the following as our conclusion:

If, as suggested in Nyambuya (2015), the Pioneer acceleration is – perhaps – due to the gravitational effect of some smeared-out material of the IPM where-in, these spacecrafts accrete from this IPM, then, Pitjeva (2010)’s preliminary null-results of the anomalous apsidal precession observations of the outer planets Uranus, Neptune and Pluto – these results do not rule out a gravitational origin of the Pioneer anomaly but rule out an even value of the Pioneer acceleration in the \([20 \text{ AU} \lesssim r \lesssim 70 \text{ AU}]\). The Pioneer acceleration may – as suggested in Nyambuya (2015); dependent on other factors such as the, mass \((m)\), mass accretion rate \((\dot{m})\) and radial velocity \((v)\) of the object in question as it swims through this supposed rarefied IPM.
References


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